STRONG ANOMALY AND CHIRAL SYMMETRY BREAKING

Bijan K. BAGCHI
Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Calcutta 700032, India

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We show, in a chiral symmetry framework, that the gluonic matrix element $\langle 0|F\tilde{F}|P\rangle$ of the η and η' is insensitive to the standard values of the quark mass parameters.

It is by now well known [1] that the presence of an anomaly term in the divergence of the flavour singlet axial current removes the axial baryon number symmetry of QCD so that strong interactions possess only a chiral SU(3) × SU(3) symmetry in the limit of the masslessness of the up, down and strange quarks. In the physical world, however, chiral SU(3) × SU(3) is only an approximate symmetry which is spontaneously broken down to SU(3), the latter being violated chiefly by the s–u and the s–d quark mass differences. Current analysis of the meson mass spectrum shows that the quark mass ratio m_s/m ($m = m_u = m_d$) takes on values close to ≈25 [2] and ≈6 [3] depending on whether the matrix elements of type $\langle 0|v_P|P\rangle$ or $\langle 0|v_P|P\rangle/(m_a + m_b)$ [(a, b) are the quark contents of the pseudoscalar meson P] transform as simple SU(3) entities, the v_i's being the standard pseudoscalar densities that belong to the $(3, \bar{3}) + (\bar{3}, 3)$ representation of chiral SU(3) × SU(3).

Recently, Goldberg [4] has shown that a careful analysis of the Ward identities combined with a fit to the 2γ decays of π^0, η and η' implies that the coupling of the η and η' to two gluons is substantial. An important consequence of this result is that the gluonic matrix elements $\langle 0|F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a|\eta, \eta'\rangle$ of the η and η' obtained from the Ward identities lead to a good quantitative understanding of the relative rates [‡1] $(\psi \to \eta'\gamma)/(\psi \to \eta\gamma)$ and also of the non-suppression of the $\psi' \to \psi\eta$ decay mode. The purpose of this note is to examine the sensitivity of the gluonic matrix elements $\langle 0|F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a|\eta, \eta'\rangle$ to the values of the quark mass ratios quoted earlier. It will be seen that these matrix elements are quite insensitive to the values of the quark mass parameters.

Assuming isospin to be an exact symmetry [‡2] for simplicity, the symmetry-breaking hamiltonian density can be written as [7]

$$H = c_0 u^0 + c_8 u^8, \tag{1}$$

where the scalar densities u_i ($\equiv i\bar{q}\lambda_i q$) along with v_i ($\equiv i\bar{q}\gamma_5\lambda_i q$) belong to the $(3, \bar{3}) + (\bar{3}, 3)$ representation of chiral SU(3) × SU(3) and c_0 and c_8 may be expressed in terms of the mechanical quark masses as

[‡1] The amplitudes for the OZI violating decays $\psi \to (\eta', \eta)\gamma$ are postulated to be proportional to the gluonic matrix elements. The process considered is $\psi \to 2g\gamma \to (\eta', \eta)\gamma$.

[‡2] By keeping an isospin violating term explicit in the symmetry breaking hamiltonian density, Gerard et al. [5] and Lahiri and Gautam [6] have shown that the non-conservation of the flavour singlet axial vector current due to the presence of the anomaly term leads to a quantitative explanation of the branching ratios $\Gamma(\psi \to \eta'\gamma)/\Gamma(\psi \to \eta\gamma)$, $\Gamma(\psi' \to \psi\pi^0)/\Gamma(\psi' \to \psi\eta)$ and the amplitude ratio $A(\eta' \to \eta 2\pi)/A(\eta \to 3\pi)$.

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$$c_0 = 6^{-1/2}(2m + m_s), \qquad c_8 = 3^{-1/2}(m - m_s). \tag{2}$$

The expressions for the divergence of the axial currents are

$$\partial_\mu A^i_\mu = \tfrac{1}{2}\mathrm{i}\bar{q}\gamma_5 \{\lambda^i, M\} q + \delta_{i0}A, \tag{3}$$

where A is the anomaly term and is given by $A = (3/2)^{1/2}(g^2/16\pi^2)\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$. In particular, one has

$$\partial_\mu A^8_\mu = (2/3)^{1/2}c_8 v_0 + 3^{-1/2}(\sqrt{2}\, c_0 - c_8)v_8, \qquad \partial_\mu A^0_\mu = (2/3)^{1/2}c_0 v_0 + (2/3)^{1/2}c_8 v_8 + A, \tag{4}$$

for the octet and the singlet states. It may be noted that these states mix to give the physical η and η' states as

$$\phi_8 = \cos\theta\, \phi_\eta + \sin\theta\, \phi_{\eta'}, \qquad \phi_0 = -\sin\theta\, \phi_\eta + \cos\theta\, \phi_{\eta'}, \tag{5}$$

$\theta \approx -10.4°$ being the standard mixing angle parameter [5].

As mentioned earlier, the matrix elements $\langle 0|v_\mathrm{P}|\mathrm{P}\rangle$ receive two kinds of contributions:

(a) The quark mass term dominating [3] $\langle 0|v_\mathrm{P}|\mathrm{P}\rangle$ in which case one has relations of the following type

$$\langle 0|v_{1+i2}|\pi^-\rangle = 2am, \qquad \langle 0|v_{4+i5}|\mathrm{K}^-\rangle = a(m + m_s), \qquad \langle 0|v_8|8\rangle = (\sqrt{2}/3)a(m + 2m_s),$$

$$\langle 0|v_8|1\rangle = \langle 0|v_8|8\rangle = \tfrac{2}{3}a(m - m_s), \qquad \langle 0|v_0|1\rangle = (\sqrt{2/3})a(2m + m_s), \tag{6}$$

where a is a constant. Eqs. (6) may be derived [8] by reducing the pseudoscalar meson P occurring in $\langle 0|v_\mathrm{P}|\mathrm{P}\rangle$, then saturating the resulting vacuum matrix elements such as $\langle \bar{u}u\rangle_0$ with single-quark intermediate states.

(b) The $\langle 0|v_\mathrm{P}|\mathrm{P}\rangle$'s transforming simply as SU(3) symmetric quantities and thus leading to

$$\langle 0|v_{1+i2}|\pi^-\rangle/\sqrt{2} = \langle 0|v_{4+i5}|\mathrm{K}^-\rangle/\sqrt{2} = \langle 0|v_8|8\rangle = \langle 0|v_0|1\rangle, \tag{7}$$

with the remaining matrix elements being zero. This case corresponds to the standard GMOR [7] scheme of hadron symmetry breaking.

In the infinite momentum frame hadronic model [9], the contributions in eq. (6) or eq. (7) correspond to whether the v_i's annihilate the valence quarks of the meson without changing the direction of the momentum or change the quantum numbers and the direction of momentum of one of the valence quarks, the vacuum absorbing the resulting pair.

Thus one can express $\langle 0|v_\mathrm{P}|\mathrm{P}\rangle$ as a linear combination of two different contributions given by eqs. (6) and (7) as

$$\langle 0|v_{1+i2}|\pi^-\rangle = 2am + b, \qquad \langle 0|v_{4+i5}|\mathrm{K}^-\rangle = a(m + m_s)b, \qquad \langle 0|v_8|8\rangle = (\sqrt{2/3})a(m + 2m_s) + b/\sqrt{2}, \tag{8, 9, 10}$$

$$\langle 0|v_8|1\rangle = \langle 0|v_0|8\rangle = \tfrac{2}{3}a(m - m_s), \qquad \langle 0|v_0|1\rangle = (\sqrt{2/3})a(2m + m_s) + b/\sqrt{2}, \tag{11, 12}$$

where a and b are to be determined from suitable inputs. While eqs. (8) and (9) lead to

$$m^2_\pi/2m^2_\mathrm{K} = \{m(b + 2ma)/[b(m + m_s) + a(m + m_s)^2]\}f_\mathrm{K}/f_\pi, \tag{13}$$

eqs. (10)–(12) give on using eq. (4),

$$f_0 m^2_0 = (2/3)^{1/2}c_0[(\sqrt{2/3})a(2m + m_s) + b/\sqrt{2}] + \tfrac{2}{3}(2/3)^{1/2}c_8 a(m - m_s) - \sin\theta\, A_\eta + \cos\theta\, A_{\eta'}, \tag{14}$$

$$f_8 m^2_8 = \tfrac{2}{3}(2/3)^{1/2}ac_8(m - m_s) + [(\sqrt{2}c_0 - c_8)/\sqrt{3}]\,[(\sqrt{2/3})a(m + 2m_s) + b/\sqrt{2}], \tag{15}$$

where m_0 and m_8 are the singlet and octet masses and A_η and $A_{\eta'}$ are defined as $A_\eta \equiv \langle 0|A|\eta\rangle$, $A_{\eta'} \equiv \langle 0|A|\eta'\rangle$. It may be noted that A_η can be expressed in terms of $A_{\eta'}$ in terms of the known $\psi \to \eta\gamma$ and $\psi \to \eta'\gamma$ rates as [4]

$$\Gamma(\psi \to \eta'\gamma)/\Gamma(\psi \to \eta\gamma) = (k_{\eta'}/k_\eta)^3 |A_{\eta'}/A_\eta|^2 \tag{16}$$

$$= 5.75 \pm 1.42 \quad \text{(experimental value)}, \tag{17}$$

so that

$$A_{\eta'} = lA_\eta, \tag{18}$$

where $l = \pm 2.66$.

We now solve for A_η and $A_{\eta'}$ from eqs. (14) and (18) in terms of a, b, quark masses, decay constants and mixing angle. Then solve for a and b from eqs. (13) and (15) in terms of quark masses and decay constants. We shall then have one equation for A_η and another for $A_{\eta'}$ in terms of quark masses, decay constants and mixing angle θ. The equations are

$$A_\eta = (l\cos\theta - \sin\theta)^{-1}[f_0 m_0^2 - (2\sqrt{2}/3)a(c_0^2 + c_8^2) - 3^{-1/2} b c_0], \tag{19}$$

$$a = (3/\sqrt{2}) f_8 m_8^2 / [2c_8^2 + (\sqrt{2}c_0 - c_8)^2 + k^2(2c_0^2 - c_8^2)], \tag{20}$$

$$b = \sqrt{6}k^2(\sqrt{2}c_0 + c_8) f_8 m_8^2 / [2c_8^2 + (\sqrt{2}c_0 - c_8)^2 + k^2(2c_0^2 - c_8^2)], \tag{21}$$

$$k^2 = \tfrac{1}{2}[4f_K m^2 m_K^2 - f_\pi m_\pi^2 (m_s + m)^2] / [f_\pi m m_\pi^2 (m_s + m) - 2f_K m^2 m_K^2],$$

which when combined yield,

$$A_\eta = \frac{f_\pi}{l\cos\theta - \sin\theta}\left(m_0^2 - \frac{2m_8^2(1+c^2)}{2c^2 + (\sqrt{2}-c)^2 + k^2(2-c^2)} - \frac{\sqrt{2}k^2 m_8^2(\sqrt{2}+c)}{2c^2 + (\sqrt{2}-c)^2 + k^2(2-c^2)}\right), \tag{22}$$

where we have assumed $f_3 \approx f_8 \approx f_0 \equiv f_\pi$ and taken $c = c_8/c_0$. It is needless to say that $A_{\eta'}$ is given by $A_{\eta'} = lA_\eta$.

In order to solve for A_η and $A_{\eta'}$ from the above expressions, we have used the values of m_0^2 and m_8^2. These values are taken, to leading order, as [5]

$$m_0^2 = m_\eta^2 + m_{\eta'}^2 - m_8^2 = m_\eta^2 \sin^2\theta + m_{\eta'}^2 \cos^2\theta = 0.897\ (\text{GeV})^2, \qquad m_8^2 = 0.321\ (\text{GeV})^2. \tag{23}$$

In table 1 are displayed the positive values of A_η and $A_{\eta'}$ for various values of c and equivalent quark mass ratios.

It is obvious from the table that both A_η and $A_{\eta'}$ are insensitive to the input values of m_s/m which have been

Table 1
Values of A_η and $A_{\eta'}$.

c	m_s/m	$A_\eta/f_\pi m_\eta^2$	$A_{\eta'}/f_\pi m_{\eta'}^2$
$-\sqrt{2}$	∞	0.87	0.76
−1.35	64.1	0.98	0.86
−1.30	35.2	0.91	0.80
−1.25	23.8	0.92	0.81
−1.2	17.8	0.92	0.81
−1.15	14.1	0.92	0.81
−1.1	11.5	0.92	0.81
−1.0	8.2	0.92	0.81
−0.95	7.1	0.92	0.81
−0.9	6.2	0.92	0.81
−0.85	5.5	0.92	0.81
−0.8	4.9	0.92	0.81

allowed to range between $5 < m_s/m < \infty$. As a matter of fact, the values of A_η and $A_{\eta'}$ are identical both for $m_s/m \approx 25$, which corresponds to the GMOR scheme and $m_s/m \approx 6$, which is the value proposed in other schemes that assume quark mass domination of $\langle 0|\upsilon_P|P\rangle$. It may be also mentioned in passing that the values of A_η and $A_{\eta'}$ obtainedhere are in agreement with the estimates of Goldberg [4].

Note added. After this paper was submitted for publication, a work of Aizawa et al. [10] on the radiative processes of η and η' mesons came to our notice. These authors have shown that the ratio of gluonic matrix elements of η and η' are insensitive to a wide variation of the ratio $\langle 0|c\bar{c}|0\rangle/\langle 0|u\bar{u}|0\rangle$. However, their model is quite different in spirit from the one adopted in this paper.

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