MORE ON MESON DECAYS OF GLUEBALLS

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In this letter, we observe that if the E(1440) is identified as a glueball with pseudoscalar quantum numbers then its decay width can be significantly large, about as broad as that of the ρ meson, if not more. However, the possibility of a narrow width cannot be rigorously ruled out. We also examine the sensitivity of the decay widths to the off-shell corrections as well as to the sign of the intermediate coupling constants.

By taking into account the mixings of a pseudoscalar glueball [identified with the recently observed E(1440) meson] with η and η' through their anomalous couplings, Senba and Tanimato [1] have recently estimated the widths of several meson-decay modes of the E(1440) meson. The purpose of this letter is to point out that these authors have not made full use of their basic equations and that if one sets up a self-consistent scheme, a definite conclusion on the total width of the E meson is not possible at present.

We begin by writing down the identity obtained in ref. [1]

$$A_E m_E^2 \Phi_E = A_\eta m_\eta^2 \Phi_\eta + A_{\eta'} m_{\eta'}^2 \Phi_{\eta'}, \qquad (1)$$

where the couplings are defined as

$$\langle 0|\tfrac{2}{3}(3\alpha_s/4\pi)\, G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a|P\rangle = A_P m_P^2, \quad P = \eta, \eta', \qquad (2)$$

$$\langle 0|\tfrac{2}{3}(3\alpha_s/4\pi)\, G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a|E\rangle = A_E m_E^2, \qquad (3)$$

in the usual framework of QCD. Given eq. (1) and recalling that $M \to M_1 + M_2$ (M a meson) the decay amplitude may be written as [2]

$$\langle M_1 M_2|M\rangle = -i(k^2 - M^2)\int d^4x\, e^{-ikx}\langle M_1 M_2|\Phi_M(x)|0\rangle, \qquad (4)$$

it is straightforward to obtain an amplitude relation of the following form

$$A_E m_E^2 \langle M_1 M_2|E\rangle = A_\eta m_\eta^2 \langle M_1 M_2|\eta\rangle + A_{\eta'} m_{\eta'}^2 \langle M_1 M_2|\eta'\rangle. \qquad (5)$$

In the above equation proper off-shell corrections have been accounted for by taking, following Cicogna [3], the corrected (c) matrix elements as

$$\langle M_1 M_2|P\rangle_c = (m_\pi^2/m_P^2)\langle M_1 M_2|P(0)\rangle. \qquad (6)$$

Considering now the tow cases corresponding to $M_1 = \delta$, $M_2 = \pi$ and $M_1 = \Psi$, $M_2 = \gamma$, we get the following pair of equations [‡1]

$$A_E m_E^2 g_{E\delta\pi} = A_\eta m_\eta^2 g_{\eta\delta\pi} + A_{\eta'} m_{\eta'}^2 g_{\eta'\delta\pi}, \qquad (7)$$

$$A_E m_E^2 g_{E\Psi\gamma} = A_\eta m_\eta^2 g_{\eta\Psi\gamma} + A_{\eta'} m_{\eta'}^2 g_{\eta'\Psi\gamma}, \qquad (8)$$

relating A_η and $A_{\eta'}$ to A_E. Eq. (8), as we shall see below, puts a useful constraint on the topological charges $A_\eta, A_{\eta'}$ and A_E. For example, noting that the ratio of $\Psi \to P\gamma$ to $\Psi \to E\gamma$ decay widths can be written as

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[‡1] It may be mentioned that it is the use of eq. (8) simultaneously with eq. (7) that makes our calculations different from the ones performed in ref. [1].

$$\Gamma(\Psi\to P\gamma)/\Gamma(\Psi\to E\gamma)$$

$$= (A_P m_P^2/A_E m_E^2)^2 \times \text{(phase space)}, \qquad (9)$$

eq. (8) implies the following sum rule

$$A_E^2 m_E^4 = A_\eta^2 m_\eta^4 + A_{\eta'}^2 m_{\eta'}^4, \qquad (10)$$

which may be used effectively to eliminate A_E between eqs. (7) and (10). However, before we do this, let us note that eq. (10) predicts

$$A_\eta/A_E = 2.4 \quad \text{and} \quad A_{\eta'}/A_E = 2.1, \qquad (11)$$

after using the experimental data [4] $A_{\eta'}/A_\eta = 0.9 \pm 0.1$ as an input. Moreover, eq. (11) gives

$$\Gamma(\Psi\to E\gamma)/\Gamma(\Psi\to\eta'\gamma)$$

$$= (A_E/A_{\eta'})^2 (m_E^4/m_{\eta'}^4) \times \text{phase space} = 1/1.35, \qquad (12)$$

in good agreement with the Crystal Ball data [5]

$$\Gamma(\Psi\to\eta'\gamma) \leqslant 1.9\,\Gamma(\Psi\to E\gamma).$$

Coming back to eq. (10), we find that if we rewrite eq. (7) as

$$g_{E\delta\pi} = g_{\eta\delta\pi} \qquad (13)$$

$$\times\,[A_\eta m_\eta^2/A_E m_E^2 + (g_{\eta'\delta\pi}/g_{\eta\delta\pi}) A_{\eta'} m_{\eta'}^2/A_E m_E^2],$$

then eq. (10) implies

$$g_{E\delta\pi}^2 = g_{\eta\delta\pi}^2 [A_\eta m_\eta^2 + (g_{\eta'\delta\pi}/g_{\eta\delta\pi}) A_{\eta'} m_{\eta'}^2]^2$$

$$\times\,(A_\eta^2 m_\eta^4 + A_{\eta'}^2 m_{\eta'}^4)^{-1}, \qquad (14)$$

with $g_{E\delta\pi}^2$ related to $\Gamma(E\to\delta\pi)$ as ‡2

$$\Gamma(E\to\delta\pi) = 3 g_{E\delta\pi}^2 |\vec{q}|/8\pi m_\delta^2. \qquad (15)$$

We now proceed to calculate the decay width of E $\to \delta(880)\pi$ and assume, with Senba and Tanimato, the pole-model [6] value of $g_{\delta\eta'\pi}^2/g_{\delta\eta\pi}^2 \approx 0.6$. Two cases need to be considered corresponding to $g_{\delta\eta'\pi}/g_{\delta\eta\pi} = \pm(0.6)^{1/2}$, respectively. For the + sign we find that $\Gamma(E\to\delta\pi) \approx 180$ MeV from eqs. (14) and (15) indicating that the decay width of the E(1440) can be considerably large – more than that of the ρ meson while for the – sign, $\Gamma(E\to\delta\pi)$ is ~20 MeV, roughly of the same order as obtained by Carlson et al. [7]. Thus it is seen that the width of the E(1440) is rather sensitive to the sign of the couplings of the intermediate states and as such a definite conclusion with regard to its width cannot be made at the moment.

The values of A_η/A_E and $A_{\eta'}/A_E$ obtained in eq. (11) also enable us to predict the $\eta\pi\pi$ and 3π decay widths of the E:

$$\langle\pi^+\pi^-\eta|E\rangle = (A_{\eta'} m_{\eta'}^2/A_E m_E^2)\langle\pi^+\pi^-\eta|\eta'\rangle$$

$$+ (A_\eta m_\eta^2/A_E m_E^2)\langle\pi^+\pi^-\eta|\eta\rangle, \qquad (16)$$

$$\langle\pi^+\pi^-\pi^0|E\rangle = (A_\eta m_\eta^2/A_E m_E^2)\langle\pi^+\pi^-\pi^0|\eta\rangle$$

$$+ (A_{\eta'} m_{\eta'}^2/A_E m_E^2)\langle\pi^+\pi^-\pi^0|\eta'\rangle, \qquad (17)$$

which turn out to be

$$\Gamma(E\to\eta\pi\pi) = 2.1\ \text{MeV}, \quad \Gamma(E\to\pi^+\pi^-\pi^0) = 0.4\ \text{keV}, \qquad (18)$$

respectively.

So far we have been using Cicogna's form of the correction factors for off-shell extrapolations. However, in view of the absence of more detailed information, we may as well use the following form of the corrected matrix element

$$\langle M_1 M_2|P\rangle_c = [\beta/(\beta + M^2)]\,\langle M_1 M_2|P(0)\rangle, \qquad (19)$$

β being a free parameter. The motivation for this form of parametrisation comes from the generalised vector dominance model where to account for the mass extrapolation effects in the intermediate off-shell vector mesons, a similar choice for the correction factor is made ‡3. Corresponding to eq. (19) we now obtain the following set of equations

$$A_E(\beta + m_E^2) g_{E\delta\pi}$$

$$= A_\eta(\beta + m_\eta^2) g_{\eta\delta\pi} + A_{\eta'}(\beta + m_{\eta'}^2) g_{\eta'\delta\pi}, \qquad (20)$$

$$A_E(\beta + m_E^2) g_{E\Psi\gamma}$$

$$= A_\eta(\beta + m_\eta^2) g_{\eta\Psi\gamma} + A_{\eta'}(\beta + m_{\eta'}^2) g_{\eta'\Psi\gamma}, \qquad (21)$$

which give on eliminating β

‡2 Note that E is an isoscalar thereby giving a factor of 3 in the rhs of eq. (15).

‡3 For a detailed discussion on this as well as on other possible forms of correction factors, see Nandy et al. [8].

Table 1
Predictions for $\Gamma(E \to \delta\pi)$. Note that Z stands for $B(E \to K\bar{K}\pi)$. In our calculations we have taken the exp. value of $\Gamma(\delta \to \eta\pi)$ = 52 MeV as an input.

$g_{\eta'\delta\pi}/g_{\eta\delta\pi}$	Z					
	0.5	0.6	0.7	0.8	0.9	1
$+(0.6)^{1/2}$	365.3	267.3	199.4	150.2	113.6	85.8
$-(0.6)^{1/2}$	1.3	9.3	21.8	37.1	54.3	72.8

Table 2
Predictions for the parameters A_η/A_E, $A_{\eta'}/A_E$ and the decay widths $\Gamma(E \to \pi\pi\eta)$, $\Gamma(E \to \pi^+\pi^0\pi^-)$.

	Z					
	0.5	0.6	0.7	0.8	0.9	1
A_η/A_E	2	2.2	2.4	2.6	2.7	2.9
$A_{\eta'}/A_E$	1.9	1.95	2.1	2.25	2.4	2.5
$\Gamma(E \to \pi\pi\eta)$ (MeV)	1.7	1.8	2.1	2.4	2.7	2.9
$\Gamma(E \to 3\pi)$ (keV)	0.3	0.32	0.38	0.45	0.49	0.56

$$\left(\frac{g_{E\delta\pi}}{g_{\eta\delta\pi}} - \frac{A_\eta}{A_E} - \frac{A_{\eta'}}{A_E}\frac{g_{\eta'\delta\pi}}{g_{\eta\delta\pi}}\right) \times \left(\frac{A_\eta}{A_E} m_\eta^2 + \frac{A_{\eta'}}{A_E} m_{\eta'}^2 \frac{g_{\eta'\delta\pi}}{g_{\eta\delta\pi}} - m_E^2 \frac{g_{E\delta\pi}}{g_{\eta\delta\pi}}\right)^{-1}$$
$$= \left(\frac{g_{E\Psi\gamma}}{g_{\eta\Psi\gamma}} - \frac{A_\eta}{A_E} - \frac{A_{\eta'}}{A_E}\frac{g_{\eta'\Psi\gamma}}{g_{\eta\Psi\gamma}}\right) \times \left(\frac{A_\eta}{A_E} m_\eta^2 + \frac{A_{\eta'}}{A_E} m_{\eta'}^2 \frac{g_{\eta'\Psi\gamma}}{g_{\eta\Psi\gamma}} - m_E^2 \frac{g_{E\Psi\gamma}}{g_{\eta\Psi\gamma}}\right)^{-1} . \quad (22)$$

In order to solve for $g_{E\delta\pi}$, we need to know $g_{E\Psi\gamma}/g_{P\Psi\gamma}$ or equivalently A_E/A_P. Since nothing beyond the following information [9]

$$B(\Psi \to E\gamma)B(E \to K\bar{K}\pi) = (3.6 \pm 1.4) \times 10^{-3} \quad (23)$$

is available at present, we have varied $B(E \to K\bar{K}\pi)$ ($\equiv Z$) from 0.5 to 1 to predict $\Gamma(E \to \delta\pi)$. The results are displayed in table 1 corresponding to $g_{\eta'\delta\pi}/g_{\eta\delta\pi} = \pm(0.6)^{1/2}$. It is clear that $\Gamma(E \to \delta\pi)$ is not insensitive to the value of $B(E \to K\bar{K}\pi)$ and seems to range between 85 MeV $< \Gamma <$ 365 MeV for $g_{\eta'\delta\pi}/g_{\eta\delta\pi} = +(0.6)^{1/2}$ and 1.3 MeV $< \Gamma <$ 72 MeV for $g_{\eta'\delta\pi}/g_{\eta\delta\pi} = -(0.6)^{1/2}$, respectively. The corresponding values of A_η/A_E, $A_{\eta'}/A_E$ along with our predictions [‡4] for the decay widths of the three-body modes $E \to \eta\pi\pi$ and $E \to 3\pi$ are listed in table 2.

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[‡4] After making similar assumptions for the coupling constants as in ref. [1].

References

[1] K. Senba and M. Tanimoto, Phys. Lett. 106B (1981) 215.
[2] B.L. Ioffe, Sov. J. Nucl. Phys. 29 (1979) 827;
A. Lahiri et al., Lett. Nuovo Cimento 29 (1980) 433;
A. Lahiri and B. Bagchi, Phys. Lett. 101B (1980) 287.
[3] G. Cicogna, Lett. Nuovo Cimento 26 (1979) 235.
[4] R. Partridge et al., Phys. Rev. Lett. 44 (1980) 712.
[5] This result has been quoted by: C. Rosenzweig, A. Salomone and J. Schechter, Phys. Rev. D24 (1981) 2545.
[6] A. Bramon and E. Masso, Phys. Lett. 93B (1980) 65.
[7] C.E. Carlson, J.J. Coyne, P.M. Fishbane, F. Gross and S. Meshkov, Phys. Lett. 99B (1981) 353.
[8] A. Nandy, B. Bagchi and S. Ray, J. Phys. G4 (1978) 989.
[9] D. Scharre et al., Phys. Lett. 97B (1980) 329;
S.L. Wu, DESY 81-003 (1981).