MASSLESSNESS OF THE UP QUARK AND $\eta \to 3\pi^0$ DECAY

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We explore new implications for the $m_u = 0$ hypothesis and show that the decay rate for $\eta \to 3\pi^0$ is in remarkable agreement with the experiment.

Considerable attention has been focussed recently on the question of existence of a light, $I = 0$, pseudoscalar meson, the axion, in connection with P and T conservation in quantum chromodynamics and gauge theories of the weak interaction [1–6]. While spontaneous breaking of a global U (1) chiral symmetry leads to a neutral boson with mass of the order 100 keV to 1 MeV [1], it is generally agreed that if the axion does not exist, one of the consequences is the vanishing of the "up" quark mass since the phase angle associated with instanton effects can then be taken to be any value whatever and therefore set to 0 thereby conserving P and T in strong interactions [1,2]. Although doubts have been raised about the usefulness of the $m_u = 0$ scheme [1,3], Deshpande and Soper have observed that one can get a remarkably accurate prediction of the $\delta(980)$ mass in this hypothesis [4]. Further, Zepeda has shown that, using the $K^+ - K^0$ mass difference as an input, $m_u = 0$ implies an improvement of the SU(2) × SU(2) symmetry [5]. In this note we report on another remarkable prediction of the $m_u = 0$ hypothesis, viz. the long standing problem of the decay rate for $\eta \to 3\pi^0$, and conclude that the massless quark hypothesis cannot be summarily rejected, especially in view of the absence of any experimental confirmation in favour of the axion.

In the usual quark model where the symmetry breaking hamiltonian density is given by [7]

$$H = u_0 + du_3 + cu_8 , \tag{1}$$

where d and c are symmetry breaking parameters and u_i along with v_i ($i = 0, 1, ..., 8$) belong to the $(3, \bar{3}) + (\bar{3}, 3)$ representation of chiral SU(3) × SU(3), the one boson to vacuum matrix elements of

$$\partial^\mu A^i_\mu = -\mathrm{i}[Q^i_A, H] , \tag{2}$$

(where the A and Q_A are axial vector currents and charges) lead to a $K^+ - K^0$ mass difference

$$(\Delta m^2_K)_{u_3} = m^2_\pi \frac{m_u - m_d}{m_u + m_d} \frac{f_\pi}{f_K} \frac{\langle 0|v_K|K\rangle}{\langle 0|v_\pi|\pi\rangle} . \tag{3}$$

When $m_u = 0$, eq. (3) becomes

$$(\Delta m^2_K)_{u_3} = -m^2_\pi \frac{f_\pi}{f_K} \frac{\langle 0|v_K|K\rangle}{\langle 0|v_\pi|\pi\rangle}, \tag{4}$$

and the naive assumption that $\langle 0|v_\pi|\pi\rangle \approx \langle 0|v_K|K\rangle$ leads to the rather large $K^+ - K^0$ mass splitting observed by Weinberg and others. If, however, we take the view that the experimentally observed $K^+ - K^0$ mass difference is more fundamental, then eq. (4) gives us the ratio

$$\langle 0|v_K|K\rangle / \langle 0|v_\pi|\pi\rangle = 0.36 , \tag{5}$$

and leads to the strange to down quark mass ratio [5]

$$m_s/m_d = 1 - m^2_{K^0}/(\Delta m^2_K)_{u_3} = 46.8 , \tag{6}$$

and

$$c|_{m_u=0} = \frac{1}{\sqrt{2}} \frac{m_d - 2m_s}{m_d + m_s} = -1.37 . \tag{7}$$

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If the massless u-quark turns out to be meaningful, it is expected that for $m_u = 0$ the u_3 term, and therefore the ratios in eqs. (5), (6) and (7), will reflect the largest deviations from the usual models. In order to test this feature we need to investigate some process, such as $\eta \to 3\pi^0$, where the u_3 term is important using a hamiltonian in which the u_3 term appears as naturally as possible.

Now, it was shown by Oakes [8] some time ago that the symmetry breaking hamiltonian density

$$\mathcal{H} = -u_0 - cu_8 , \tag{8}$$

when rotated about the 7th axis in the SU(3) space through an angle 2θ and with strangeness conservation imposed upon the result, attained the form

$$\mathcal{H}' = -u_0 + \sqrt{2}(1 - \tfrac{3}{2}\sin^2\theta)\, u_8 + \sqrt{\tfrac{3}{2}}\sin^2\theta\, u_3 , \tag{9}$$

with $c = -\sqrt{2}$. By postulating that eq. (9) represents the hadron hamiltonian density the following relation was derived:

$$\sin^2\theta = \tfrac{1}{3}\sqrt{2}(c + \sqrt{2}) , \tag{10}$$

from which θ turned out to be in very good agreement with the Cabibbo angle. The remarkable feature about eq. (9) is the presence of the u_3 term which breaks not only chiral SU(2) × SU(2) but also isospin invariance.

Given eq. (9), the decay rate for $\eta \to 3\pi^0$, neglecting terms of order m_π^2 and relativistic corrections in the kinematics, can be expressed as [9]

$$\Gamma(\eta \to 3\pi^0) = \frac{\sqrt{3}}{9216\pi^2}\left(\frac{\sin\theta}{f_\pi}\right)^4 m_\eta^3 (m_\eta - 3m_\pi)^2 . \tag{11}$$

In the $m_u = 0$ case where $c = -1.37$, the value of θ turns out to be 8.3° from eq. (10), about half of the Cabibbo angle. Substituting this value in eq. (11) and calculating for the decay rate we get $\Gamma(\eta \to 3\pi^0) = 0.27$ keV for $f_\pi \approx 94$ MeV in remarkable agreement with experiment [10].

To conclude, such agreements as obtained in this case and in the example studied by Deshpande and Soper make the $m_u = 0$ theory at least phenomenologically rather attractive, if theoretically demanding some reservations.

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