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Comment on "From $N=2$ supersymmetry to quantum deformation"

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We consider the possibility of reducing the recently proposed $N=2$ deformed supersymmetric Hamiltonian to the standard form

Recently, Beckers and Debergh have developed an interesting link between $N=2$ supersymmetry and quantum deformation [1] To this end they have proposed the following forms of two hermitian (odd) supercharges

$$Q_1 = \frac{-iM\partial_x + RW}{\sqrt{2}}, \quad Q_2 = \frac{-iN\partial_x + SW}{\sqrt{2}}, \qquad (1)$$

where W is the superpotential and the matrices M, N, R, S are restricted to the choices $M=\sigma_1$, $N=\sigma_2$, $R=a\sigma_1+b\sigma_2$, $S=a\sigma_1-B\sigma_2$, σ_1 and σ_2 being the usual Pauli matrices The parameters a and b satisfy the constraints $a^2+b^2=1$, a, $b\in\mathbb{R}$, $b\geqslant 0$ One should realize that superpotential $W(x)$ is associated with the ground-state wave function so that the latter can be normalized [2]

The supersymmetric Hamiltonian (even) is defined through $H^{SS}_{ab}=Q_1^2=Q_2^2$ leading to

$$H^{SS}_{ab} = -\tfrac{1}{2}\partial_x^2 + \tfrac{1}{2}W'^2 + \tfrac{1}{2}bW''\sigma_3 - ia(W'\partial_x + \tfrac{1}{2}W''), \qquad (2)$$

where the prime implies a partial derivative with respect to x In contrast the standard Witten's Hamiltonian has the underlying structure [3]

$$H^{SS}_{\text{Witten}} = -\tfrac{1}{2}\partial_x^2 + \tfrac{1}{2}W'^2 + \tfrac{1}{2}W''\sigma_3 \qquad (3)$$

It may be seen that (3) follows from (2) on setting $a=0$ and $b=1$

The purpose of this comment is to point out that the deformed Hamiltonian $H^{SS}_{a,b}$ can always be brought to Witten's form by gauging away the superfluous terms in (2)

To justify our point let us consider making a replacement $i\partial_x \to i\partial_x + aW'$ in (1) and consequently in (2) It is easy to verify that such a shift not only drives away the first derivative term in (2) but reduces H^{SS}_{ab} to the standard form

$$H^{SS}_{b} = -\tfrac{1}{2}\partial_x^2 + \tfrac{1}{2}b^2W'^2 + \tfrac{1}{2}bW''\sigma_3 \qquad (4)$$

Indeed a correspondence with (3) may be established by scaling $W \to bW$

For the harmonic oscillator case considered in ref [1] where $W=\frac{1}{2}wx^2$, the deformed energy-levels expectedly emerge as b times the energy-levels induced by (3) Should we choose to work with the form (4), the eigenfunctions of the oscillator system turn out to be simply proportional to $\exp(-ibwx^2/2)\times H_n(\sqrt{bw}\,x)$

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References

[1] J Beckers and N Debergh, Phys Lett B 286 (1992) 290
[2] A Lahiri, P K Roy and B Bagchi, Intern J Mod Phys A 5 (1990) 1383
[3] E Witten, Nucl Phys B 188 (1981) 513