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Chiral and $U_A(1)$ phase transitions in QCD: an analysis of the $I=0$ pseudoscalars

Surajit Sen [1], B. Bagchi [2]

Department of Applied Mathematics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India

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Abstract

We study chiral and $U_A(1)$ phase transition in the dilute gas approximation. A comprehensive analysis of the mass and mixing angel of η and η' reveals that in the transition of the quark-gluon-plasma droplets to the hadrons, besides the chiral phase transition, the $U_A(1)$ phase transition also plays a significant and non-trivial role.

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1. Introduction

The phase transition in quantum chromodynamics (QCD) [1], apart from its ample scope of wide application in the astrophysics of a dense neutron star, is expected to predict some observable signals of Quark-Gluon-Plasma (QGP), albeit transiently, in the future ultra-relativistic heavy ion collision experiments. Formally, the study of the phenomenon of the phase transition involves two steps: (i) the complete knowledge of the global symmetry exhibited by the model and (ii) the behaviour of the order parameter with temperature which characterizes the order of the transition. The QCD, apart from its hidden local colour $SU_C(3)$ gauge symmetry, exhibits a rich global flavour symmetry, namely $SU_V(3) \times SU_A(3) \times U_V(1) \times U_A(1)$. Because of the baryon number conservation and the isospin symmetry, $U_V(1)$ and $SU_V(3)$ symmetries remain exact while the remaining two are broken. In particular $SU_A(3)$ is spontaneously broken due to the formation of the quark-antiquark bound state called the quark condensate. The condensate, which is believed to be dissociated from the bound state above a critical temperature, is a good order parameter to characterize the chiral phase transition (ChPT). In recent times there has been a spate of activities in studying the ChPT in QCD using different approaches [2–8].

It should be emphasized that the physical picture of the axial-vector or $U_A(1)$ symmetry retoration at a high temperature is quite complicated due to our

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[1] Permanent address: Department of Physics, G C College, Silchar 788 004, India.

[2] E-mail: bbagchi@cucc.ernet.in

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incomplete understanding of the $U_A(1)$ symmetry. The $U_A(1)$ symmetry of the QCD is broken by the axial-vector anomaly term which enables us to account for the absence of any massless singlet state in the spectrum of the pseudoscalar mesons. Although instanton physics provides a good recipe of solving the so called $U_A(1)$ problem [9], the latter still remains an unsettled issue [10–12]. There now exist some impressive results on the chiral and deconfinement transition [13–15]; however, the UPT has not

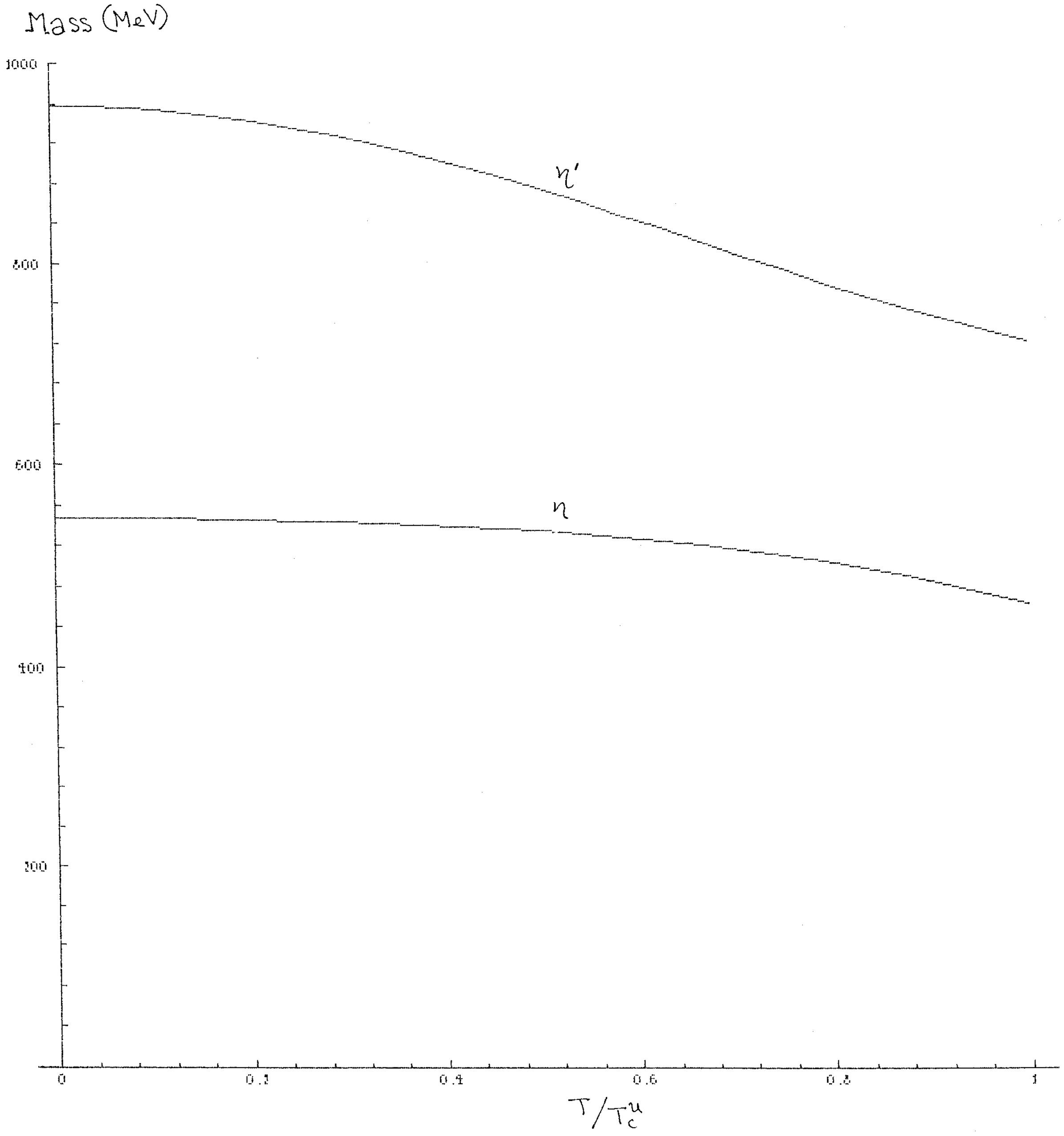

Fig. 1. Variation of η and η' mass with T/T_c^u.

drawn much attention so far due to the elusive nature of the $U_A(1)$ symmetry.

Based on the dilute instanton gas model, more than a decade ago Pisarsky and Wilczek [16] conjectured the possibility of restoring the $U_A(1)$ symmetry at high temperatures. It is gratifying to see that subsequent few investigations of this issue have underlined the crucial role of the UPT in addition to the ChPT [17–21]. The whole situation may be summarized as follows. In the hadronization of the QGP-

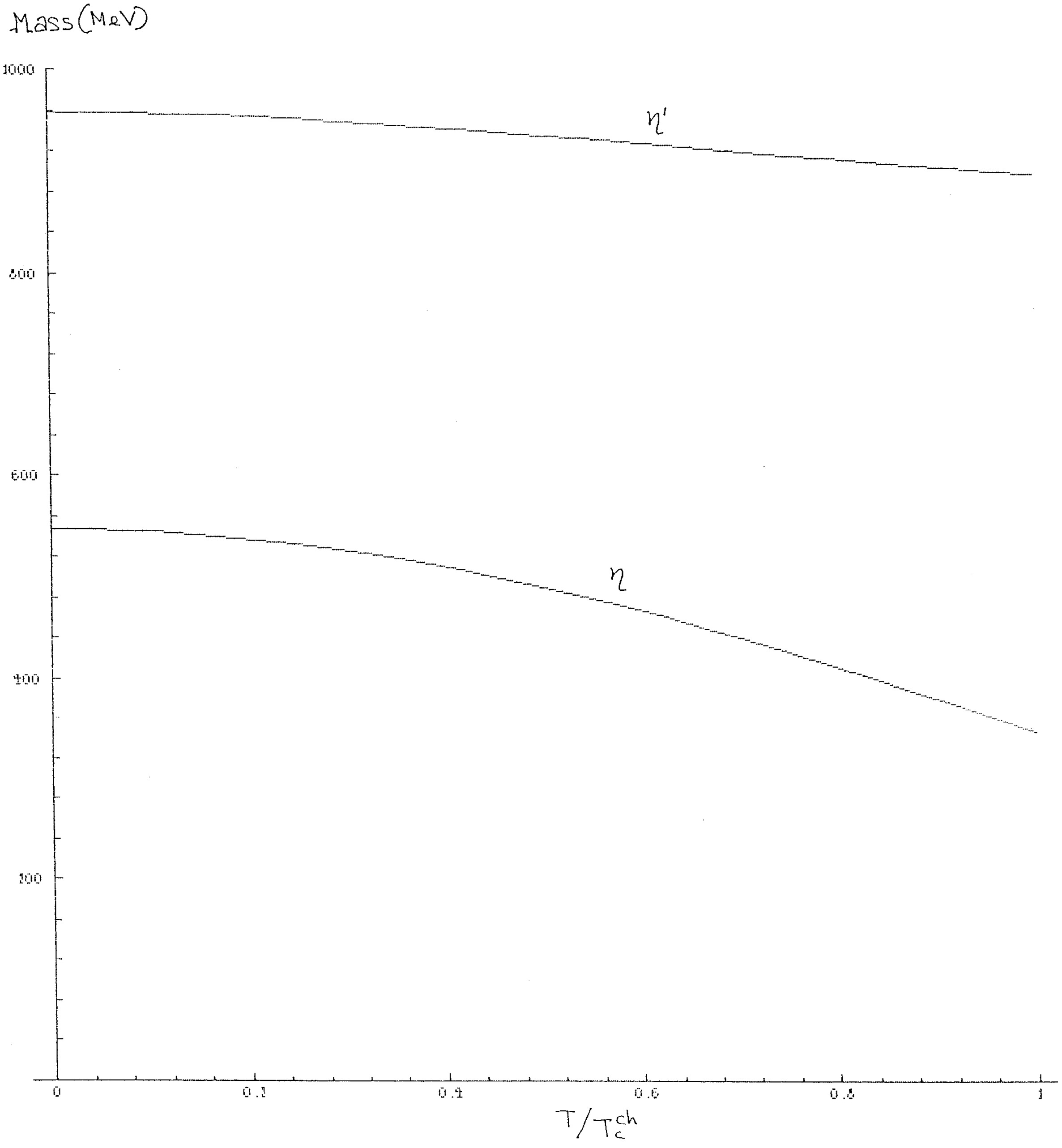

Fig. 2. Variation of η and η' mass with T/T_c^{ch}.

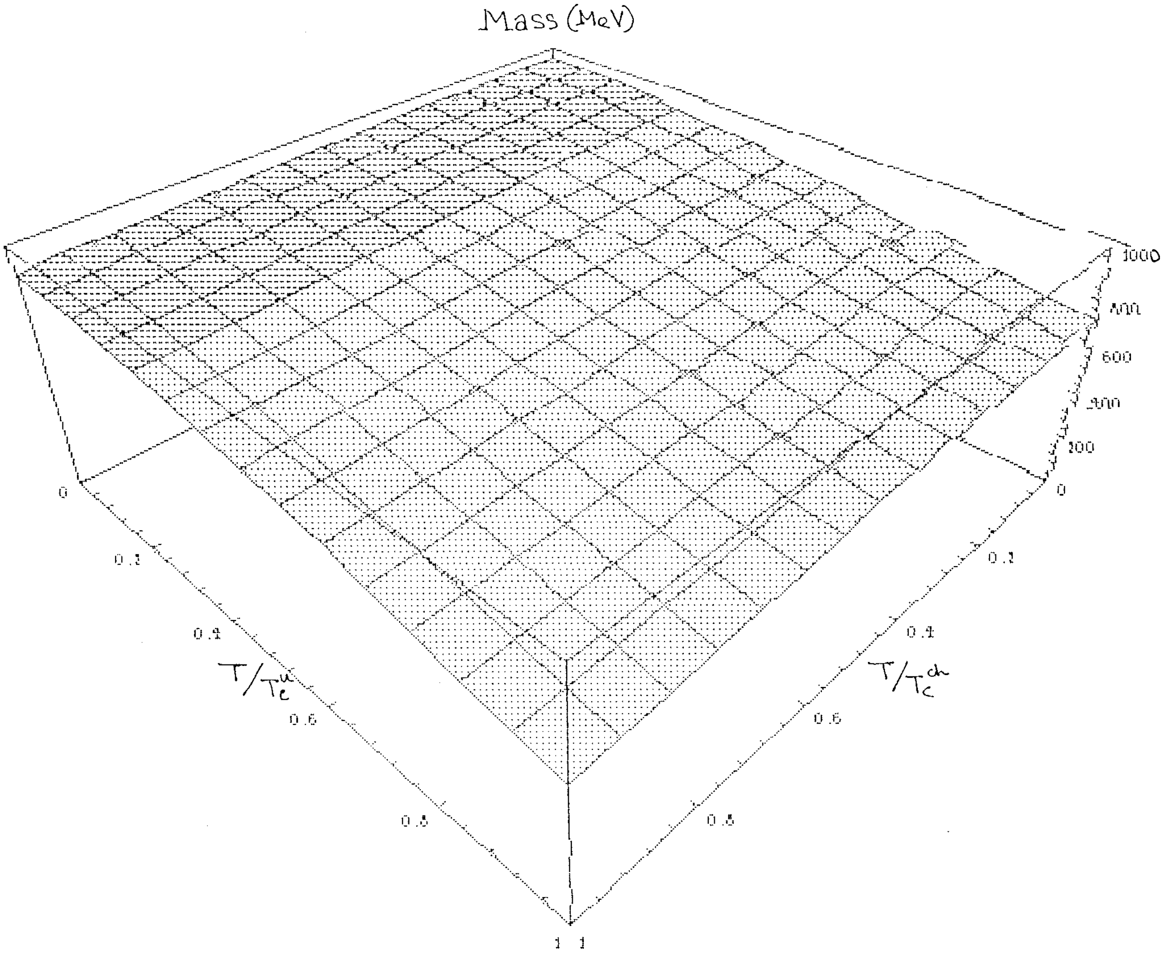

Fig. 3. Variation of η' mass with T/T_c^{ch} and T/T_c^u.

droplet, the plausible routes of the chiral and $U_A(1)$ symmetry restoration are:

Scenario-I: UPT preceding earlier than ChPT ($T_c^{ch} > T_c^u$):

$$SU_V(3) \times U_V(1)$$
$$\overset{T_c^u}{\Rightarrow} SU_V(3) \times U_A(1) \times U_V(1)$$
$$\overset{T_c^{ch}}{\Rightarrow} SU_V(3) \times SU_A(3) \times U_V(1) \times U_A(1)$$

Scenario-II: ChPT preceding $U_A(1) PT (T_c^{ch} < T_c^u)$:

$$SU_V(3) \times U_V(1)$$
$$\overset{T_c^{ch}}{\Rightarrow} SU_V(3) \times SU_A(3) \times U_V(1)$$
$$\overset{T_c^u}{\Rightarrow} SU_V(3) \times SU_A(3) \times U_V(1) \times U_A(1)$$

While Pisarky and Wilczek have advocated for the first route, other groups have supported the second one [18,20]. In principle there can also be a third possibility in which there exists no critical temperature indicating the absence of any latent heat associated with these transitions. In the context of scenarios I and II, the third scenario is in-between, both critical temperatures being the same.

The purpose of the article is to examine the effects of the chiral and $U_A(1)$ transition on the mass spectrum and mixing angle of the isoscalar η and η' mesons. The choice of such a system is natural since the axial-vector anomaly plays a crucial role in explaining the large mass of the η' meson. The remaining sections of the paper are organised as follows. In Section 2 we develop the necessary methodology required to address to the issues mentioned above. In Section 3 we discuss the numerical results. The concluding section deals with the outcome and implications of our investigation.

2. Methodology

The (mass)^2 matrix of the $\eta_8-\eta_0$ system is given by

$$\begin{bmatrix} M_{88}^2 & \xi M_{80}^2 \\ \xi M_{08}^2 & \xi^2 M_{00}^2 \end{bmatrix} := \frac{1}{F_8^2} \times \begin{bmatrix} -\frac{1}{3}(m_u+m_d+4m_s)\upsilon & -\frac{\sqrt{2}}{3}(m_u+m_d-2m_s)\upsilon \\ \frac{\sqrt{2}}{3}(m_u+m_d-2m_s)\upsilon & -\frac{2}{3}(m_u+m_d+m_s)\upsilon+\chi^2 \end{bmatrix} \tag{1}$$

where M_{ij} $(i,j=0.8)$ are the matrix elements with m_i $(i=u,d$ and $s)$, ξ $(=F_0/F_8)$ and υ $(=\langle uu\rangle=\langle dd\rangle=\langle ss\rangle)$ defining the mass of the current quarks, nonet breaking parameter and quark condensate respectively. The anomaly induced term χ^2, which can also be identified with topological susceptibility [22,23], contributes to the singlet channel only. Assuming nonet symmetry ($\xi=1$), the physical masses of η and η' mesons can be fitted using the inputs $m_u=5$ MeV, $m_d=10$ MeV, $m_s=200$ MeV, $F_8(=F_\pi)=93$ MeV with $\upsilon\approx-(193\text{ MeV})^3$ and $\chi^2/F_8^2\approx(871\text{ MeV})^2$. The $\eta-\eta'$ mixing angle turns out to be $|\theta|\approx15°$.

The finite temperature effects can be incorporated by parametrizing $\upsilon\to\upsilon(T)$ and $\chi^2/F_8^2\to K(T)$ where the decay constant F_π is taken to be temperature independent. In the dilute gas approximation (DGA), the instantons may be considered as very weakly interacting with $K(T)\approx K(0)\exp\{-(T/T_c^{u})^2\}$ [21,24,25]. Similarly for the quarks, the asymptotic freedom favours the possibility of a large fermi surface about which they are almost freed or interacting very weakly through a colour channel [6,25]. The DGA is a fairly good approximation and we can write $\upsilon(T)=\upsilon(0)\exp\{-(T/T_c^{ch})^2\}$ in good accord with the expectation of chiral perturbation theory [26]. With the above choices of parametrization, the temperature dependent masses of η and η' mesons may be readily obtained by diagonalising Eq. (1).

$$\begin{aligned} &M_{\eta',\eta}\left(T/T_c^{ch},T/T_c^{u}\right) \\ &\quad=\left[\tfrac{1}{2}\left\{K-0.0637\upsilon\right.\right. \\ &\qquad\left.\left.\pm\left(0.0027\upsilon^2-0.061\upsilon K+K^2\right)^{1/2}\right\}\right]^{1/2} \end{aligned} \tag{2}$$

where $\upsilon=\upsilon(T/T_c^{ch})$ and $K=K(T/T_c^{u})$. Further, the unitarity constraint of the mass matrix causes the $\eta-\eta'$ mixing angle to be also temperature dependent in a manner

$$\begin{aligned} &\left|\tan\theta\left(T/T_c^{ch},T/T_c^{u}\right)\right| \\ &\quad=\left[\left\{M_{88}^2(T)-M_\eta^2(T)\right\}\right. \\ &\qquad\left./\left\{M_{\eta'}^2(T)-M_{88}^2(T)\right\}\right]^{1/2} \end{aligned} \tag{3}$$

We now proceed to solve Eqs. (2) and (3) and discuss the effects of the ChPT and UPT on the mass spectrum and mixing angle of the η and η' mesons.

3. Numerical results

To start with we consider two extreme limits of the critical temperatures: $T_c^{ch}\gg T_c^{u}$: In this case the chiral symmetry is assumed to be restored much later than the $U_A(1)$ symmetry. This implies that the chiral condensate is not sensitive to temperature below T_c^{u}, but the anomaly induced term K is. This situation is very similar to the observations of Pisarsky and Wilczek. Nevertheless, to put our work in the proper perspective, we would like to mention some additional features. Fig. 1 shows the variation of η and η' masses with temperature where mass of the latter one is least affected at lower temperatures. At the critical temperature T_c^{u}, their masses are reduced to $M_{\eta'}(0,1)\approx723$ MeV and $M_\eta(0,1)\approx463$ MeV respectively so that $M_{\eta'}(0,0)-M_\eta(0,0)>M_{\eta'}(0,1)-M_\eta(0,1)$. In other words, when $U_A(1)$ symmetry is restored the zero temperature numerical sum rule $M_{\eta'}(0,0)-M_\eta(0,0)\approx2.1m_s$ gets modified to $M_{\eta'}(0,1)-M_\eta(0,1)\approx1.3m_s$. Variation of the $\eta-\eta'$ mixing angle with temperature leads to the substantial enhancement of the mixing value from $|\theta(0,0)\approx$

15° to $|\theta(0,1)| \approx 39°$. This shows that as the temperature starts approaching T_c^u, the singlet (octet) admixture in $\eta(\eta')$ meson gets more and more pronounced. It must however be pointed out that the present route is based on an exponential suppression factor in the instanton density which, as the work of Ref. [27] has shown, undergoes no modification at small temperatures. Lattice data have also indicated [28] insensitivity of topological susceptibility right up to the critical value of the temperature. Evidently these results effectively rule out this scenario.

$T_c^{ch} \ll T_c^u$: In this case the chiral symmetry is restored much earlier than the $U_A(1)$ symmetry so that the chiral condensate becomes the function of temperature below T_c^{ch} only. The masses of η' and η shown in Fig. 2 are reduced to $M_{\eta'}(1,0) \approx 898$ MeV and $M_\eta(1,0) \approx 346$ MeV leading to $M_{\eta'}(1,0) - M_\eta(1,0) \approx 2.8 m_s$. Thus we find an opposite behaviour in contrast to the previous case in that $M_{\eta'}(0,0) - M_\eta(0,0) < M_{\eta'}(1,0) - M_\eta(1,0)$. Further, the mixing angle now reduces to $|\theta(1,0)| \approx 5°$ indicating that η behaves as almost pure $SU(3)$ octet while η' as a pure $SU(3)$ singlet state.

The existence of critical temperature in the UPT and ChPT shows that they are of either first or higher order transition. The corresponding mass spectrum in such cases is expected to follow any one of the two routes mentioned above with a discontinuity at their critical temperature. If the two critical temperatures of these phase transitions are compara-

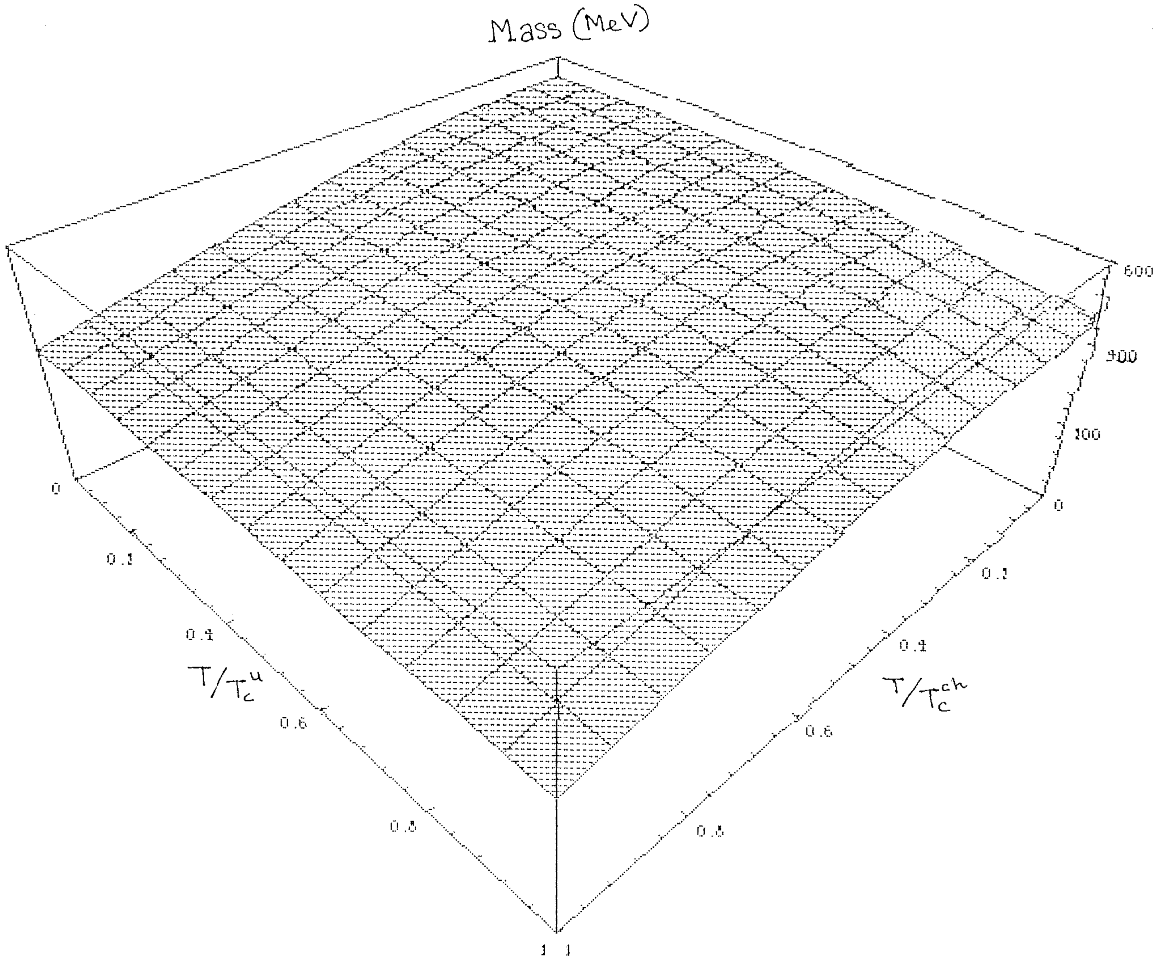

Fig. 4. Variation of η mass with T/T_c^{ch} and T/T_c^u.

ble to each other then the whole situation is quite different. Figs. 3 and 4 gives the three dimensional depiction of all plausible routes through which masses of η' and η mesons drop to global minima. In the first scenario if the UPT precedes the ChPT ($T_c^{ch} > T_c^u$), we note that the mass gap of $\eta'(\eta)$ becomes $M_{\eta'}(0,0) - M_{\eta'}(0,1) \approx 1.2m_s(M_\eta(0,0) - M_\eta(0,1) \approx .4m_s)$ at T_c^u followed by further reduction $M_{\eta'}(0,1) - M_{\eta'}(1,1) \approx .7m_s(M_\eta(0,1) - M_\eta(1,1) \approx .7m_s)$ at T_c^u. On the other hand in the second scenario when the ChPT occurs earlier than the UPT ($T_c^{ch} < T_c^u$), then $\eta'(\eta)$ mass reduces as $M_{\eta'}(0,0) - M_{\eta'}(1,0) \approx .3m_s(M_\eta(0,0) - M_\eta(1,0) \approx m_s)$ at T_c^{ch} and then $M_{\eta'}(1,0) - M_{\eta'}(1,1) \approx 1.6m_s(M_\eta(1,0) - M_\eta(1,1) \approx .1m_s)$ at T_c^u. We thus find that in either of the routes, the net change of mass is $M_{\eta'}(0,0) - M_{\eta'}(1,1) \approx 1.9m_s$ and $M_\eta(0,0) - M_\eta(1,1) \approx 1.1m_s$ respectively indicating that none of the mesons becomes a Goldstone mode even after both phase transitions. Furthermore, it is worth noting that in both the scenario the gluonium enriched η' meson loses its substantial mass during UPT while quarkonium enriched η mass loses maximum mass during ChPT. The plots also show that if none of the phase transitions involve critical temperatures the mass spectrum η and η' show smooth passage to global minima. The mixing angle curve in Fig. 5 does not exhibit any change even after both the transitions through any of the

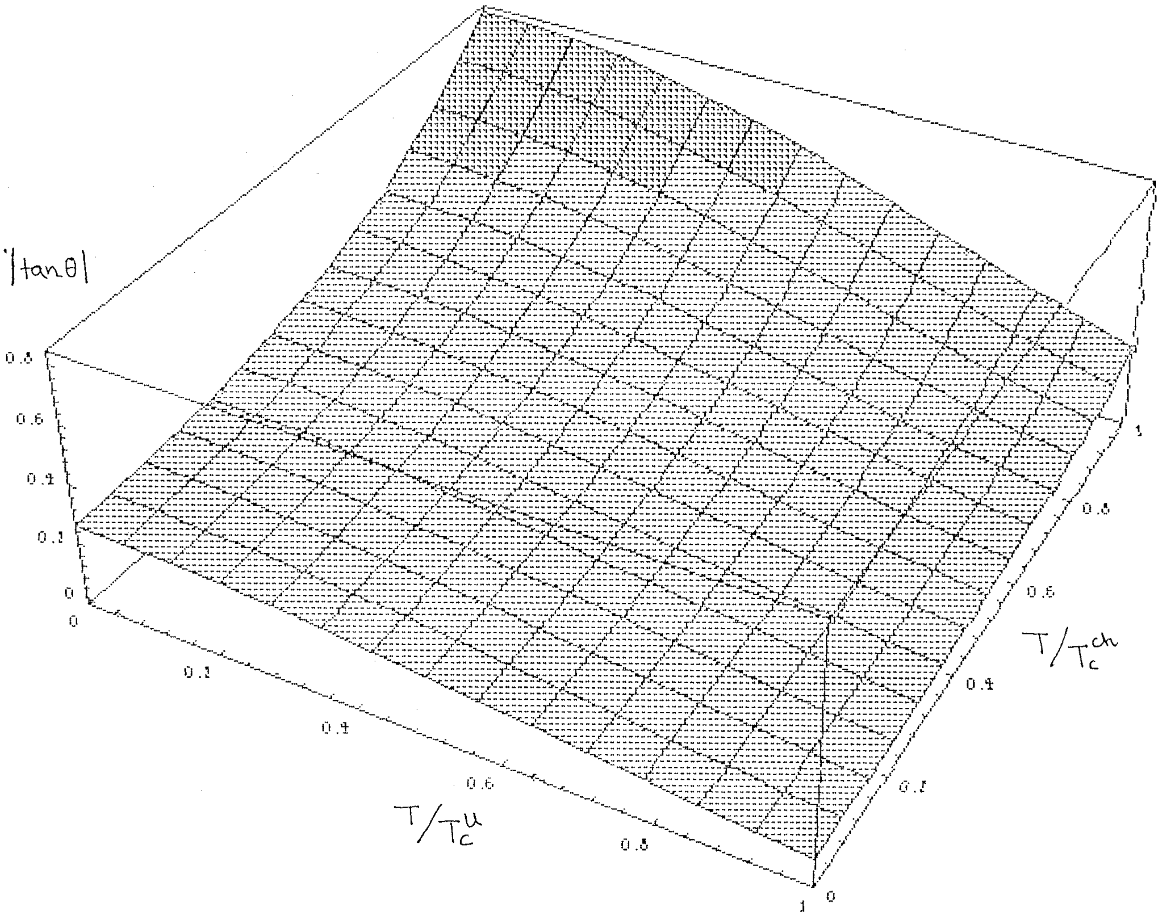

Fig. 5. Variation of $|\tan\theta|$ with T/T_c^{ch} and T/T_c^u.

routes indicating that the proportion of pure η_8 and η_0 state remains the same in η and η' mesons.

4. Conclusion

In this paper we have analysed the effects of the chiral and the $U_A(1)$ phase transition on the masses and mixing angle of η and η' mesons and have pointed out the significant role of the latter in the hadronisation of the QGP droplet. Our treatment, which considers ChPT and UPT on an equal footing, relies on the diagonalisation of the $(\mathrm{mass})^2$ matrix of $\eta - \eta'$ system in the dilute gas approximation. An artifact of the routes considered is that the η' mass always remains higher than η even after the phase transition. A more consistent picture is likely to emerge if the effects of the thermal loop is taken into consideration. It is shown that the mass spectrum of the singlet η' meson distinctly differs from the octet η meson indicating the non-trivial role of the axial-vector anomaly in the QCD phase transition. We have also discussed the possible routes through which the hadronisation proceeds and have distinctive nature of different routes. Further studies, especially from the thermodynamical consideration, is required to assess which of these routes is energetically most favourable. The temperature dependence of the pseudoscalar mixing angle is a natural outcome of the present analysis. The $\eta - \eta'$ mixing in the instanton model was also discussed in detail by Schaefer and Schafer [29]. Admittedly the complicated instanton induced four-fermion interaction may improve our calculations. However we have confined our interest primarily within the simplest forms of the model. Finally about the DGA it must be remarked that as the instanton density goes smaller at high temperatures and the gas becomes more dilute, the interaction between them cannot become negligible: otherwise chiral symmetry would never be really restored. It may be noted that instanton provide an effective determinantal interaction between the left and right-handed quarks which can generate a dynamical quark mass [30]. In conclusion, the $U_A(1)$ phase transition in addition to the chiral phase transition may be within reach the future colliders like LHC and RHIC since both of them have separated the hardronic phase from the quark-gluon phase.

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