TEMPERATURE DEPENDENT GRAVITATIONAL CONSTANT AND BLACK HOLE PHYSICS

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This letter examines the thermodynamics of black holes in case of a varying gravitational constant. We find that the second law of thermodynamics may remain valid contrary to the opinions expressed by some.

Following the idea of spontaneous symmetry breaking, there have been various attempts [1] to introduce a term involving a coupling between a scalar field (ϕ) and the curvature scalar (R) in the lagrangian. Thus Linde [2], by including an interaction term of the form $\sim R\phi^2$ obtained the modified field equations [‡1]

$$R_{ik} - \tfrac{1}{2} g_{ik} R = -8\pi G_e T_{ik}, \tag{1}$$

with the effective gravitational constant G_e as

$$G_e^{-1} = G_0^{-1} - \tfrac{4}{3}\pi\phi^2. \tag{2}$$

The above equation shows that at a certain value of ϕ, G_e blows up and for still higher values changes sign thus signalling the onset of an antigravity regime. While earlier researchers [2–5] have considered such a possibility, here we would like to point out that in view of (1), when $G_e \to \infty$, one would have a geometrical singularity shown by the blowing up of the Ricci tensor components (unless there is a simultaneous vanishing of all the components of T_{ik} which would mean a violation of the positive definite condition for the energy) so that an antigravity phase ought to be preceded by the appearance of a singularity. A special case of this is explicit in a recent calculation for the Friedman metric by Pollock [3], who finds that the stage of infinite-G_e coincides with the big-bang singularity.

‡1 In the following, all the quantum corrections due to the ϕ field are ignored and T_{ik} represents the normal energy–momentum tensor, the scalar field under consideration being a purely classical field.

Let us now examine the thermodynamics of black holes in view of (2). The second law of black hole physics viz. $\mathrm{d}A \geqslant 0$ depends primarily on the cosmic censorship hypothesis and only weakly on the field equations in that

$$-R_{ij} v^i v^j = 8\pi G_e (T_{ij} - \tfrac{1}{2} g_{ij} T) v^i v^j$$

is assumed positive definite for the arbitrary time-like vector v^i. As we have already seen that the antigravity phase is necessarily preceded by a singular state and if we confine ourselves to non-singular stages, the positive definiteness of $-R_{ij}v^i v^j$ would continue to hold good (even for a varying G_e case) and hence the area increase theorem also holds true.

For a Schwarzschild black hole the area is

$$A = 16\pi G_e^2 M^2. \tag{3}$$

Hence

$$\mathrm{d}(G_e M) = (1/32\pi)\,\mathrm{d}A/G_e M. \tag{4}$$

The area increase theorem thus requires $G_e M$ to be a non-decreasing function in any classical process. Should this condition be violated, the conclusion would be that the cosmic censorship hypothesis does not hold good and naked singularities may evolve.

The continuity equations as following from the vanishing of the divergence of the lhs of (1) are

$$(G_e T^{ik})_{;k} = 0, \tag{5}$$

If we consider a perfect fluid with energy stress tensor

$$T^i_k = (p + \rho) v^i v_k - p\delta^i_k, \tag{6}$$

eq. (5) gives

$$\{G_e[(p+\rho)v^i v_k - p\delta^i_k]\}_{;i} = 0,$$

so that the familiar equation in case of constant G

$$d(\rho\delta v) + p\, dv = 0 \qquad (7)$$

(where δv represents an increment in the volume element) now assumes the form

$$d(G_e\rho\delta v) + G_e p\, dv = 0 \qquad (8)$$

– in other words, we recover the energy conservation theorem in case of adiabatic flow with p and ρ replaced by $G_e p$ and $G_e \rho$ respectively.

In case there is an exchange of heat, eq. (8) would be modified to read

$$d(G_e E) + G_e p\, dV = d(G_e Q), \qquad (9)$$

where $d(G_e Q)$ is not an exact differential. Following the ideas of thermodynamics we write $d(G_e Q) = G_0 T \times ds$ where T, the integrating factor, is identified with temperature and the state function S is identified with entropy. Regarding the reasonableness of this choice it may be noted that for small variations in G_e, the entropy of the black hole is expected to remain fixed [‡2] Thus with the quantity $G_e M$ as an adiabatic invariant one has

$$\partial S/\partial(G_e E) = \partial S/\partial(G_e M) = 1/G_0 T, \qquad (10)$$

and consistency with eq. (4) is maintained if we make the following ansatz about entropy and temperature of a Schwarzschild black hole [‡3]

$$S = A/4\pi G_0, \qquad T = 1/8\pi G_e M. \qquad (11a, b)$$

It is not out of place to mention here the expressions for entropy and temperature used by other investigators. Davies [4,5] adopts

$$S = A/4\pi G_e, \qquad T = 1/8\pi G_e M, \qquad (12a, b)$$

and thereby finds that the second law of thermodynamics may be violated for black hole evaporation. However as pointed out by Pollock [6], Davies failed to take account of the change in the conservation relation due to the variation of G. In fact the possibility of violation of the energy principle is inherent in Davies' principle. The cosmic censorship hypothesis ensures that $dA \geqslant 0$ but with G_e varying, $d(A/G_e)$ may well be decreasing even in classical processes not involving black hole evaporation. Pollock [6] on the other hand takes

$$d(G_e E) = T\, d(G_e S) - G_e p\, dV, \qquad (13)$$

and interprets the entropy increase principle as $d(G_e S)/dt > 0$.

Returning now to eq. (11), it may be pointed out that the identification of S with the area A (11a) is essentially dictated by the area increase theorem and the natural constraint that we should have agreement with the usual theory in case of constant G. With eq. (11a) the principle of increase of entropy of the black holes in classical processes (i.e. processes not involving the emission of particles from a black hole) become identified with the area increase theorem [7].

It is also easy to see that for the case of the black holes in a radiation bath of temperature T', the rate of change of entropy is given by [‡4]

$$\begin{aligned} dS/dt &= (dS_{bh}/dt) + (dS_r/dt) \\ &= [dS_{bh}/d(MG_e)]\, d(MG_e)/dt + dS_r/dt \\ &= T^{-1}(a'T'^4 - aT^4)A + \tfrac{4}{3}(aT^3 - a'T'^3)A \\ &= A\,aT^3\{[(a'/a)y^4 - 1] + \tfrac{4}{3}[1 - (a'/a)y^3]\}. \end{aligned} \qquad (14)$$

The rhs is $\geqslant 0$ for $a'/a \leqslant 1$ and $y = T'/T > 0$, thus preserving the second law of thermodynamics.

Thus our formalism is on this point identical with that of Pollock's – only the "non-decreasing" property is attributed by him to the product of entropy and effective gravitational constant while we consider it more appropriate to call the non-decreasing function itself the entropy of the system.

We conclude with a few remarks on the evaporation process of the black holes in the present varying-G formalism. As discussed above, with $G_e M$ as an adiabatic invariant, the rate of radiation is given by

$$d(G_e M)/dt = -aG_0 T^4 A, \qquad (15)$$

where G_0 is the asymptotic value of G for $T \to 0$. Using the following relation between G_e and G_0 [4,8]

‡2 This also seems to imply that the quantity $(G_e M)$ remains an adiabatic invariant.

‡3 Note that the mass parameter M of the black hole also depends on variations in G_e.

‡4 Compare ref. [5].

$$G_e^{-1} = G_0^{-1} - \alpha T^2 \tag{16}$$

where α depends on the choice of the coupling constants, we find that eqs. (15), (11) and (3) yield

$$\Delta t = (32\pi^3/3a) G_0^2 \times \{[M + (M^2 + \alpha/16\pi^2 G_0)^{1/2}]^3 - (\alpha/16\pi^2 G_0)^{3/2}\}. \tag{17}$$

Here Δt is the time taken for the black hole to evaporate completely ($M \to 0$). As with Hawking's case [9], here too, we find that the time interval is finite and that evaporation is catastrophic in nature. However the present process of complete evaporation is slower than what was obtained by Hawking in the constant-G case.

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Note added. Instead of G_e as given by eq. (2), one may as well wish to lump the entire ϕ dependence on the energy–momentum tensor T_{ik}, the rhs of eq. (1) now reading as $-8\pi G_0 T'_{ik}(\phi)$. Such a choice does not seem to preclude an entry into the anti-gravity region owing to the possibility of the existence of a negative energy density for the ϕ field. We thank the referee for bringing this point to our attention. However with the form $T'_{ik}(\phi)$ being completely unknown it does not seem obvious that any definition of the entropy (S) and the temperature (T) of the black hole can be obtained unambiguously.

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