# Proton spin dynamics and axial singlet charge

B. Bagchi

*Department of Applied Mathematics, Vidyasagar University, Midnapore 721101, West Bengal, India*

and

S. Basu

*Department of Theoretical Physics, Indian Association for the Cultivation of Science, Calcutta 700 032, West Bengal, India*

Received 19 June 1991

We consider the nucleonic matrix element of the divergence of the gauge invariant flavour singlet axial current and show by a careful analysis that isospin-dependent terms persist in it. We also relate the axial singlet charge to the sum of the quark spin contributions and $F/D$ ratio and point out that in order to be consistent with the EMC data a small value is not ruled out.

Much work has been devoted (see for a recent review ref. [1]) towards understanding the smallness of the SU(3) singlet axial charge following the discovery of the spin effect in polarized deep inelastic μ–proton scattering. The latest EMC analysis [2] may be summarized by defining the moments $\Delta q'$ through

$$\Delta q' \cdot s_\mu = \langle p, s | \bar{q}\gamma_\mu\gamma_5 q | p, s \rangle ,$$

$s_\mu$ being the spin vector. The available data on the structure function of the polarized proton and the results on the hyperon decays lead to the estimates $\Delta u' = 0.78 \pm 0.08$, $\Delta d' = -0.47 \pm 0.08$, $\Delta s' = -0.19 \pm 0.8$, implying

$$\sum \Delta q' = \Delta u' + \Delta d' + \Delta s' = 0.12 \pm 0.24 . \tag{1}$$

It should be noted that SU(3) restricts $\Delta u' + \Delta d' - 2\Delta s' = 3F - D$ while the Bjorken sum rule fixes $\Delta u' - \Delta d' = g_A = F + D$.

The role of the axial anomaly in QCD in determining the quark spin fractions of the proton has been investigated [3–7] by a number of authors. Veneziano [4,5] has pointed out that the data on $\Delta u'$, $\Delta d'$ and $\Delta s'$, in effect, signal an approximate decoupling of the would be U(1) Goldstone boson from the nucleon. More or less, a similar conclusion has been reached by Hatsuda [6] working within a theory of large-$N$ colour chiral dynamics. Recently, Fritzsch [7]

has shown it to be possible to define flavour-independent matrix elements of the anomalous divergence which are related to the axial U(1) charge and to the spin densities inside the proton.

The purpose of this letter is to look into Fritzsch's scenario in some detail and examine the reasonableness of the claim that in the nucleonic matrix element of the divergence of the isoscalar current the isospin-dependent terms cancel exactly. Subsequently we make an effort to estimate the axial singlet charge by relating it to $\sum \Delta q'$ and $F/D$ ratio.

We begin by writing down the nucleonic matrix element of the isovector axial current,

$$\langle p | \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d | p \rangle$$
$$= \bar{u}(p) [G_1(q^2)\gamma_\mu\gamma_5 - G_2(q^2)q_\mu\gamma_5] u(p') . \tag{2}$$

The induced form factor $G_2(q^2)$ picks up a pion pole but this pole makes no contribution to the divergence at $q^2 \to 0$. Taking the divergence, one is thus left with the Goldberger–Treiman relation

$$2MG_1(0) = 2Mg_A = 2f_\pi g_{\pi NN} . \tag{3}$$

On the other hand, the isosinglet axial current between nucleons generates the following matrix element:

$$\langle p | \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d | p \rangle$$

$$= \bar{u}(p)[G_1^0(q^2)\gamma_\mu\gamma_5 - G_2^0(q^2)q_\mu\gamma_5]u(p') . \quad (4)$$

Since no Goldstone boson is associated with $G_2^0$, one has in contrast to (3)

$$2MG_1^0(0) = 2f_\eta g_{\eta NN} + C , \quad (5)$$

where $C$ represents the contribution from the continuum assuming an unsubtracted dispersion relation for the divergence form factor. In ref. [7] the magnitude of $C$ has been anticipated to be large but no estimate has been provided.

Instead, it has been shown [7] how the matrix element of the anomalous divergence cancels exactly with the matrix element of the pseudoscalar isovector quark density. To look into this, consider the divergence equations for the isovector and isosinglet axial currents,

$$\partial^\mu(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)$$

$$= (m_u + m_d)(\bar{u}\,i\gamma_5 u - \bar{d}\,i\gamma_5 d)$$

$$+ (m_u - m_d)(\bar{u}\,i\gamma_5 u + \bar{d}\,i\gamma_5 d) , \quad (6a)$$

$$\partial^\mu(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)$$

$$= (m_u + m_d)(\bar{u}\,i\gamma_5 + \bar{d}\,i\gamma_5 d)$$

$$+ (m_u - m_d)(\bar{u}\,i\gamma_5 u - \bar{d}\,i\gamma_5 d) + a , \quad (6b)$$

where $a$ is the anomaly defined by

$$a = N_f \frac{\alpha_s}{2\pi} \text{tr}\, G\tilde{G} . \quad (7)$$

Assuming PCAC, one can express

$$\langle p | (m_u - m_d)(\bar{u}\,i\gamma_5 u - \bar{d}\,i\gamma_5 d) | p \rangle$$

$$= (\bar{u}\,i\gamma_5 u)\, 2Mg_A \frac{m_u - m_d}{m_u + m_d} , \quad (8)$$

while in the singlet channel one has

$$\langle p | (m_u + m_d)(\bar{u}\,i\gamma_5 u + \bar{d}\,i\gamma_5 d) | p \rangle$$

$$= (\bar{u}\,i\gamma_5 u)2Mg_A^0 . \quad (9)$$

Now, to estimate $\langle p | a | p \rangle$, Fritzsch [7] makes use of the relation

$$\langle 0 | a | \pi^0 \rangle = \frac{m_d - m_u}{m_d + m_u} f_\pi m_\pi^2 . \quad (10)$$

In effect this means

$$\langle p | a | p \rangle = -(\bar{u}\,i\gamma_5 u)\, 2Mg_A \frac{m_u - m_d}{m_u + m_d} , \quad (11)$$

which cancels exactly with the right-hand side of eq. (8). The divergence of the isoscalar current (6b) between proton states is thus free from any isospin-dependent contamination:

$$\langle p \partial^\mu(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) | p \rangle$$

$$= (\bar{u}\,i\gamma_5 u)\, 2Mg_A^0 . \quad (12)$$

A few remarks on the derivation of eq. (10) may be in order. Eq. (10) is arrived at

(a) using the standard GMOR [8] transformation properties of the pseudoscalar densities $\bar{u}\,i\gamma_5 u$ $(\bar{d}\,i\gamma_5 d)$ and

(b) assuming that $\pi^0$ is an ideal superposition of $P_u$ and $P_d$ states i.e. $\pi^0 = (P_u - P_d)/\sqrt{2}$ where $P_u$ $(P_d)$ is the pseudoscalar bound state of $u$ and $\bar{u}$ ($d$ and $\bar{d}$).

Let us define

$$\langle 0 | \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d | \pi^0 \rangle = f_\pi P_\mu . \quad (13)$$

As a consequence

$$2\langle 0 | m_u \bar{u}\,i\gamma_5 u - m_d \bar{d}\,i\gamma_5 d | \pi^0 \rangle = f_\pi m_\pi^2 . \quad (14)$$

Since an $I = 0$ axial vector current between the vacuum and $I = 1$ state yields a vanishing matrix element we also have the condition

$$\langle 0 | \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d | \pi^0 \rangle = 0 . \quad (15)$$

Taking the divergence gives

$$\langle 0 | 2(m_u \bar{u}\,i\gamma_5 u + m_d \bar{d}\,i\gamma_5 d) + a | \pi^0 \rangle = 0 . \quad (16)$$

Expressing now $\pi^0$ as $(P_u - P_d)\sqrt{2}$ and assuming [8] that $\langle 0 | \bar{u}\,i\gamma_5 u | P_u \rangle$ is the same as $\langle 0 | \bar{d}\,i\gamma_5 d | P_d \rangle$, it is straightforward to derive eq. (10) from eqs. (14) and (15).

As noted by Fuchs [9] long ago, the trouble with eq. (10) is that in the limit $\alpha_s \to 0$, it implies $m_u = m_d$. To overcome this, it was proposed in ref. [9] that perhaps $\pi^0$ is not simply a $(P_u - P_d)/\sqrt{2}$ state but depends on the $\pi^0 - \eta$ mixing angle.

Let us parameterize the mixing by defining

$$\pi^0 = \cos\theta\, P_u - \sin\theta\, P_d ,$$

$$\eta = \sin\theta\, P_u + \cos\theta\, P_d , \quad (17)$$

where $\theta = 45°$ corresponds to the ideal situation.

If we now carry through the calculation exactly in a similar manner as performed above but replacing $\pi^0$ by the new combination of $P_u$ and $P_d$ in terms of $\theta$, we obtain

$$\langle 0|a|\pi^0 \rangle = \frac{1-\tan\theta}{1+\tan\theta} - \frac{m_u - m_d \tan\theta}{m_u + m_d \tan\theta}, \qquad (18)$$

and hence

$$\langle p|\partial^\mu(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)|p\rangle$$

$$= (\bar{u}\,i\gamma_5 u)\,2M\left[ g_A^0 + g_A \frac{m_u - m_d}{m_u + m_d} \right.$$

$$\left. - g_A \left( \frac{m_u - m_d \tan\theta}{m_u + m_d \tan\theta} + \frac{1-\tan\theta}{1+\tan\theta} \right) \right]. \qquad (19)$$

A welcome feature of eq. (18) is that unless $\tan\theta \to 1$, $m_u \neq m_d$ in the limit $\alpha_s \to 0$. In the limit $\theta \to \pi/4$ (i.e. for ideal states) however, the right-hand side of eq. (18) drops to zero for $m_u = m_d$. But then this implies $\alpha_s$ is not small since a quick inspection of eq. (18) reveals that $\theta \to 0$ is consistent with $\alpha_s \to 0$.

(Note that [9] $\tan\theta$ is $O(\alpha_s^2)$ and so vanishes for $\alpha_s \to 0$.)

The structure of eq. (19) is not a simplistic one; nevertheless one can see that there is no exact cancellation of the isospin-dependent terms in the right-hand side of eq. (19). One can argue that $\pi^0$ is dominantly a $(P_u - P_d)/\sqrt{2}$ state and so the bulk of the contribution to the right-hand side of eq. (19) comes from the right-hand side of eq. (12). But then one is at a loss to understand to logical consistency of eq. (10) in the limit $\alpha_s \to 0$.

It is instructive to follow the procedure of Cheng and Li [10]. Indeed, as we shall presently see, it does provide an estimate of $G_1^0$ employing standard current algebra techniques. First of all, we define [#1] [11] the gluon helicity component $\Delta g$ as

$$\langle p|a|p\rangle = -2M \frac{\alpha_s}{\pi} \Delta g\, \bar{u}\, \gamma_5 u . \qquad (20)$$

Then, the fraction $\Delta q$ of the proton's spin carried by the quark $q$ is not just $\Delta q'$ but actually a linear com-

bination of $\Delta q'$ and an anomalous contribution due to the anomaly term, viz. $(\alpha_s/\pi)\Delta g$:

$$\Delta q' = \Delta q - \frac{\alpha_s}{2\pi} \Delta g . \qquad (21)$$

Between proton states, the axial divergence equations involving the QCD anomaly $\partial^\mu(\bar{q}\gamma_\mu\gamma_5 q) = 2m_q \bar{q}\, i\gamma_5 q + (\alpha_s/2\pi)\,\mathrm{Tr}\, G\tilde{G}'$ give rise to [10]

$$2M\Delta u' = 2m_u \nu_u - 2M\frac{\alpha_s}{2\pi}\Delta g ,$$

$$2M\Delta d' = 2m_d \nu_d - 2M\frac{\alpha_s}{2\pi}\Delta g , \qquad (22)$$

where $\nu_q \bar{u}\,i\gamma_5 u = \langle p|\bar{q}\,i\gamma_5 q|p\rangle$ in the notation of Cheng and Li.

Further substituting eq. (6a) into eq. (2) we get at zero momentum transfer

$$2Mg_A = (m_u + m_d)(\nu_u - \nu_d)$$

$$+ (m_u - m_d)(\nu_u + \nu_d) . \qquad (23)$$

Keeping in mind the Goldberger–Treiman relation (3), one obtains from eq. (23)

$$\nu_u + \nu_d = \frac{2M}{m_u - m_d}\left( g_A - \frac{f_\pi}{M} g_{\pi NN} \right). \qquad (24)$$

Likewise, let us substitute eq. (6b) into eq. (4). It follows that

$$2MG_1^0 = (m_u + m_d)(\nu_u + \nu_d) + (m_u - m_d)(\nu_u - \nu_d)$$

$$- 4M\frac{\alpha_s}{2\pi}\Delta g . \qquad (25)$$

Inserting (24) into eq. (25) and utilizing the expressions for $\nu_u$ and $\nu_d$ from eq. (22) we get [#2,#3]

$$G_1^0 = \Delta u' + \Delta d'$$

$$= \frac{1}{3}\left( 3 - \frac{4D}{F+D} \right) g_A + \tfrac{2}{3} \sum \Delta q' , \qquad (26)$$

leading to the estimate

$$G_1^0 = 0.30 \pm 0.16 , \qquad (27)$$

---

[#1] The replacement of $\Delta q'$ by relation (21) is not unique and depends on the convention adopted [12,13]. However, for a natural way of separating the matrix element into quark and gluon components see the discussions of ref. [5].

[#2] Relation (26) does not depend on $\Delta g$ explicitly. Note that there exist controversies in the determination of the magnitude and sign of $\Delta g$, see Geng and Ng [3].

[#3] There have been attempts to relate $\Sigma\Delta q'$ to the $\eta'$–nucleon coupling and $\eta'$-decay constant, see Birse, Efremov et al., and Dorokhov and Kochelev [3].

where we have used the flavour SU(3) relationships

$$\Delta u' = \frac{1}{3}(F+D) + \frac{1}{3}\sum \Delta q',$$

$$\Delta d' = -\frac{2}{3}D + \frac{1}{3}\sum \Delta q'. \tag{28}$$

In deriving eq. (26) we have not made use of any pole dominance approximation. The uncertainty in the estimate (27) (which essentially stems from the present experimental value of $\sum \Delta q'$) is large and so a small value of $G_1^0$ is not ruled out. Comparing (26) and (5) we remark that in the above estimates we have not [7] related the axial singlet charge $G_1^0$ to the η–nucleon coupling. For one thing, in the chiral limit, gluonic leakage persists [#4] in η and so it does not become massless in that limit. For another, the continuum contribution does not vanish in the massless quark limit but, on the contrary, may assume a large value in the singlet channel.

To summarize, we have noted in this paper that:

(a) there is no exact cancellation of the isospin-dependent terms in the nucleonic matrix element of the divergence of the gauge invariant flavour singlet axial vector current, and

(b) that the axial singlet charge $G_1^0$ may be related to the sum of the quark spin contributions and $F/D$ ratio.

---

[#4] This means n–η' mixing can take place even in the chiral limit.

## References

[1] H. Fritzsch, Mod. Phys. Lett. A 5 (1990) 1815.

[2] J. Ashman et al., Nucl. Phys. B 328 (1990) 1.

[3] R. Decker, Th. Leize and M. Nowakouski, Phys. Lett. B 244 (1990) 497;
A.V. Efremov, J. Soffer and N.A. Törnqvist, Phys. Rev. Lett. 64 (1990) 1495;
B. Bagchi and A. Lahiri, J. Phys. G 16 (1990) L239;
X. Ji, Phys. Rev. Lett. 65 (1990) 408;
Riazuddin and Fayyazuddin, Phys. Rev. D 41 (1990) 3517;
C.Q. Geng and J.N. Ng, Phys. Lett. B 243 (1990) 451;
A.E. Dorokhov and N.I. Kochelev, Phys. Lett. B 245 (1990) 609;
M.C. Birse, Phys. Lett. B 249 (1990) 291.

[4] G. Veneziano, Mod. Phys. Lett. A 17 (1989) 1605.

[5] G.M. Shore and G. Veneziano, Phys. Lett. B 244 (1990) 75.

[6] T. Hatsuda, Nucl. Phys. B 29 (1990) 376.

[7] H. Fritzsch, Phys. Lett. B 242 (1990) 451.

[8] M. Gell-Mann, R.J. Oakes and B. Renner, Phys. Rev. 175 (1988) 2195.

[9] N.H. Fuchs, Phys. Lett. B 97 (1980) 151.

[10] T.P. Cheng and L.-F. Li, Phys. Rev. Lett. 62 (1989) 1441.

[11] A. Efremov and O. Teryaev, Dubna preprint JIN-EZ-88-287;
G. Altarelli and G.G. Ross, Phys. Lett. B 212 (1988) 391;
R. Carlitz, J. Collins and A. Mueller, Phys. Lett. B 214 (1988) 229;
L. Mankiewicz and A. Schafer, Phys. Lett. B 242 (1990) 455.

[12] A.V. Manohar, Phys. Rev. Lett. 65 (1990) 2511; preprint UCSD/P Th 90-20;
R. Jaffe and A.V. Manohar, Nucl. Phys. B 337 (1990) 509;
G. Bodwin and R. Qiu, Phys. Rev. D 41 (1990) 2755.

[13] U. Ellwanger and B. Stech, Phys. Lett. B 241 (1990) 409;
Z. Phys. C 49 (1991) 683.