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*Article*

# Party Formation and Coalitional Bargaining in a Model of Proportional Representation

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**Abstract:** We study a game theoretic model of a parliamentary democracy under proportional representation where ideologically motivated citizen groups form parties, voting occurs and governments are formed. We study the coalition governments that emerge as functions of the parties' seat shares, the size of the rents from holding office and their ideologies. We show that governments may be minimal winning, minority or surplus. Moreover, coalitions may be 'disconnected'. We then look at how the coalition formation game affects the incentives for party formation. In particular, we show that when the rents from office are low, the median citizen stands unopposed, and when rents are high, there is more political entry. For intermediate rents, we show that strategic dropouts can happen to influence the final policy. We show that the incentives for strategic dropout can be higher under proportional representation than plurality voting, contrary to Duverger's law. Our model explains the diverse electoral outcomes seen under proportional representation and integrates models of political entry with models of coalitional bargaining.

**Keywords:** proportional representation; party formation; coalitions

**JEL Classification:** C72; D72; H19

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## 1. Introduction

Electoral systems vary across the world. The two main ways in which votes translate into seats in a legislature are via a proportional system or via a winner-take-all system. Nearly half the democracies in the world have some form of proportional representation, whereby legislative seats for political parties are roughly proportional to the vote shares received by them. Around a third of democracies use winner-take-all systems, with over half of these countries using plurality voting where the candidate with a plurality of votes gets elected. (https://www.fairvote.org/research-electoralsystems-world accessed on 31 March 2022).

In this paper, we examine the incentives for party formation under proportional representation (PR) and compare it with plurality voting (PV). PR is one of the most widely used electoral systems. Its proponents believe it is more representative of the will of the people than PV, where seats can be highly disproportional to the votes cast. However, under PV, the splitting of votes between like-minded parties may be partly mitigated by political parties coalescing through formal or informal agreements (or even mergers of parties) or strategically dropping out of races in some constituencies. At first glance, this does not seem to be the case for PR. After all, parties do not need to drop out ex ante and can come together in parliament to form coalition governments once seat shares are known. Given this, one would expect to see more political entry under PR than under PV. Indeed, this was predicted by the French sociologist Maurice Duverger, who hypothesized that PV would mainly see a two-party system (see [1–3] for formalizations of what is called Duverger's law (see [4]), which says that plurality rule leads to two-party rule), while PR would lead to a multi-party system. However, it is not necessarily true that parties under

PR do not have strategic decisions to make on whether to enter, and indeed, we are able to illustrate the incentives for strategic dropouts under PR. More generally, we analyze how the incentives for party formation change with the magnitude of rents or political power from office compared to ideology.

There is, of course, some evidence that PV does favour two-party systems, while PR is associated with a multiparty system. (There are, however, exceptions, with India, which uses PV, having several political parties, as do the Philippines and Canada). Consequently, in democracies that use PR, it is also unusual for a single party to control more than half the seats in parliament. In a study of 313 elections in 11 democracies in Europe (see [5]) from 1945–1997, it was found that only 20 of the elections returned a single party with more than half the seats in parliament. The latest figures from the European Representative Democracy Data Archive suggests that this trend continues (see https://erdda.org/party-government-in-europe-database/, accessed on 31 March 2022). Moreover, coalitions were diverse in size (they can be minimal winning, minority or surplus) and need not be ideologically connected. Refs. [6,7] report instances of ideologically disconnected coalitions. Ref. [8] provides a theoretical explanation of this. Given the coalition formation process, it is important to understand how these results are consistent with endogenous party entry, in particular, comparing it with a stylized model of PV.

In this paper, we construct a game theoretic model of PR which endogenizes party formation, voting as well as post-election coalition politics. We use a pure form of PR where votes translate into seat shares, as this allows us to understand if there are incentives not to stand for election that do not stem from any non-proportionality, such as a minimum threshold. In stage 1 (party formation or candidate entry), parties that are made of citizen candidates with a designated ideal policy decide whether or not to incur a cost and stand for election. In stage 2, voting occurs, and in stage 3, if a party gets more than 50% of the votes, their ideal policy is implemented, and they enjoy all the rents from office. If no party gets a majority, then there is a pre-specified rule by which a party is asked to form a government; this party is called the formateur in the literature. We consider two formateur selection procedures used in the literature, one in which the largest party is asked to form a government (see [9]) and another where the probability of being asked to form a government is proportional to seat share (see [10], who provide empirical evidence that this approximates the formateur selection procedure for many European countries).

While using a formateur selection procedure to model the ex-post bargaining process, we depart from the literature by precluding the possibility of efficient bargaining (in particular, of being able to make unlimited side transfers) or the ability to be able to commit to a policy at the government formation stage. Instead we assume that the post election outcome is given by a seat weighted average of the ideal policies of the parties making up the coalition. The particular assumption about the bargaining outcome has strong empirical support. Empirical studies of power sharing among coalition partners (see [7,11]) have found substantial evidence that coalition partners share cabinet portfolios in proportion to their relative seat shares. Since a large bulk of political power is vested in various ministerial offices, the politician in charge of a particular ministry is entitled to that power as well as the right to make a policy in the relevant area.

Our second point of departure from the previous literature concerns our treatment of the status quo in the event of the failure to form a government. We assume that in the event that the attempts at government formation fail, a consensus government comprised of all the parties is formed. Under such a government, the implemented policy is the seat-weighted average of the ideal points of all the members of the legislature. Unlike the efficient bargaining approach where in equilibrium the coalitions formed do not change with changes in the relative value of rents from office compared to ideology, our approach allows us to study these changes. Further, given the post-election bargaining game, we study the incentives for parties to stand for election and thereby affect the post-election policy.

Thus, we are able to formalize the issue of how party entry and government formation are affected by the relative importance of ideology to rents from office. In doing so, we generate a number of refutable predictions about the role of policy motivation vis-a-vis political power in the determination of government formation as well as political party formation.

To understand how the incentives of political entry under PR compares with PV, we use a stylized model of PV, namely one in which all districts are identical. While stylized, it makes the comparison with the purest form of PR with the *worst* type of PV. Allowing for heterogeneity across districts would have allowed for more parties as different parties may be *stronger* in different districts and allowed for a multi-party PV. We want to examine if there is the possibility of a multi-party PV even without such considerations. In doing so, we integrate the 'citizen-candidate' approach as in [12,13] with a variant of the coalition formation literature.

Our main results are as follows: we find that there exists an equilibrium of the political game which has the median group (when it is unique) being the only group to stand for elections, and under some configurations, we show that this could in fact be the unique equilibrium. Hence, this is contrary to the Duvergerian prediction (see [4]) that PR promotes more party formation than does PV. This occurs under the purest form of PR and without any heterogeneity across districts under PV. Instead, this comes from the fact that parties can drop out in an election under PR to influence the final policy that is chosen. This is true under PV as well but in a close election, there may be incentives to enter in PV as there is a probability of getting a big majority as well that would not happen under PR. We further show how political competition can increase with increases in the value placed on rents from office. In particular, beyond a certain value of the rents, we get a (non-unique) political equilibrium where all the ideological groups contest for elections. We also check the robustness of our results with respect to the formateur selection procedure. In particular, when using the formateur selection in order of rank, as proposed by [9], we find similar results: at low rents, the median party stands uncontested as a unique equilibrium outcome. However, a formateur selection model mimicking a parliamentary procedure is necessary for our results since, if policies are simply determined by a majority rule game, at most two parties would form.

The next section discusses some more papers which are related to our work. This is followed by presenting the model, solving the legislative game and then solving the entire political game.

## 2. Related Literature

Our work is related to several strands in the literature, particularly with several papers on coalitional bargaining, party positioning and endogenous party formation. It also draws on the insights provided by classic works on electoral systems, the activist literature on PR (see [14], for example) and several case studies which throw light on actual coalitional structure.

The study of what type of coalitions will form in equilibrium dates back to Riker (see [15]). However, the concern in Riker's work is with the division of a fixed 'pie' which members of the winning coalition are entitled to. Hence, he predicts a *minimum winning coalition*, i.e., the minimal winning coalition made of the smallest number of members (for some of the theoretical papers on coalition formation both with a constant pie as well as a non-constant sum pie, see [16] and the references in the survey by [17]. In [18,19] the role of a party when the space is multidimensional rather than uni-dimensional is analyzed. For surveys on coalitions see [20–22]. For a survey of different voting procedures in the context of political elections see [23]). When ideology is considered, there are no longer compelling reasons to predict a minimal winning coalition. Instead, following [24], the natural thing to predict would be a 'minimum winning connected' coalition, i.e., a coalition that does not leave out a partner who is in between two coalition partners on the ideological dimension. There is a large theoretical body of literature (see [25–29], for instance). Until the papers by [5,30], which we collectively refer to as BDM (for their authors Baron, Diermeier and Merlo), most of the theoretical papers did make such predictions. Our paper also generates the coalitional diversity seen in BDM, but unlike BDM, we do not assume parties can make unlimited transfers to each other. There are also a fair number of models of party formation/strategic entry starting from the 'citizen candidate' models of endogenous candidate entry to papers by [31–33] (in these papers, a distinction is made between a candidate and a party). These papers either assume that the winner is selected by plurality rule or they model the post-election policy outcome as a majority rule game, thus missing one of the most important aspects of 'hung' parliaments (which is very common under PR) which involves coalitional bargaining to form a government. Thus, our model adds to the literature on party formation by adding a detailed bargaining stage post-election. There are also papers dealing with strategic entry under more general outcome functions most notably [34], but the generality of the paper does not allow them to generate any sharp predictions except that under complete information, at least one player (candidate) will behave strategically.

The fact that PR promotes diversity has been made by [35]. The paper assumes that the policy is a weighted average of the two-parties policy with the weights being proportional to seat shares. The main aim is to provide an explanation of why policies pursued might deviate from the median voter's position. Ref. [36] integrate this approach with the citizen candidate literature, and [37] consider a model with non-strategic parties but strategic voters. Both papers predict two parties under proportional representation contrary to empirical evidence. Moreover, the assumption of a vote-weighted average does not capture the institutional details of coalition formation and government policy making, which is an important aspect of democracies under PR, which we explicitly model.

A more complete analysis of the electoral process has been made in [9,30,38,39]. Of these, Ref. [30] also notes that with 'strategic voting', minority parliaments may form in equilibrium, and voters do not always vote for the party closest to their ideal point. The first result is of interest as it provides a justification for dealing with the coalition formation stage under a minority parliament. Unlike our paper, these do not deal with party formation, and their parties are only endogenous in the sense that they choose positions. Ref. [40] does consider how legislative bargaining affects the incentives of parties, but their main concern is with whether or not a given set of parties (in their model three) will enter into pre-electoral alliances, and if so, with whom. Ref. [41] is another paper that considers a party formation model that has similarities with our paper in doing a rich analysis of parliamentary bargaining. Contrary to our model (as they themselves point out), rents are linked to parliamentary participation, while in our model it is linked to government participation. This leads to interesting differences in results. While we believe most of the rents form office flow from being in government, in the real world, rents do accrue from both. Hence, we can think of their paper as complementary to this work. Other papers considering party formation under assumptions made by [30] includes [42].

In summary, we differ from these papers in two major ways. First, we make different assumptions about coalitional bargaining (in particular, by assuming no commitment and non-transferable utility). Second, we integrate the 'institution-free' citizen-candidate approach to politics with the rich institutional details of parliamentary democracy. This allows us to understand how political groups decide whether or not to contest elections based on both their ability to be part of government as well as (through strategic drop outs) to influence the final policy when they are not likely to be in government.

## 3. Model

In this section, we formalize the political process under PR. We denote by $\mathcal{N} = \{1, 2, \cdots, N\}$ the set of groups of citizens in the polity where $1 < N < \infty$. Let $N_i$ denote the measure of citizen belonging to group $i$. Let $X$ denote the policy space, which we take to be a compact subset of the real line. The payoff of a (representative) citizen belonging to group $i$ is denoted by

$$-u_i(|x_i - x|) + T_i \tag{1}$$

where $x \in X$ is the policy implemented, $x_i$ is the ideal policy of party $i$, and $T_i$ is the transfer of money received by the citizen. The interpretation of $T_i$ is the amount of directed transfer made to that group as opposed to the policy which has a public good interpretation. We shall throughout assume that a group which is a part of the government receives a transfer $\lambda P$, where $\lambda$ is the relative size (seat share) of the group in terms of the number of parties in the government. This is the main distinction from being in government and out of government. Only members of a government are able to hold ministries and hence make directed transfers to their own groups. This is the particular way we interpret the rents from office, and following the strong empirical evidence (as discussed in the Introduction), we assume the ministries (which we assume is vested with the ability to make these transfers) to be split according to party size (note that we could alternately have assumed that people care for policy and rents (or directed transfers) in the ratio $\alpha$ and $1-\alpha$. Hence, the payoff for a citizen of group $i$ can be written as $-\alpha u(|x_i - x|) + (1-\alpha)T_i$. The comparative statics that we do would have been in terms of the marginal rate of substitution between policy benefits and rents. This makes no qualitative change, so we keep the simpler formulation). We assume that $u(0) = 0; u' > 0; u'' \geq 0$.

The political process can be described as a four stage game: 1. candidate entry, 2. elections, 3. government formation, and 4. policy selection. We describe these stages in detail below. This game will then be solved using backwards induction to find its sub-game perfect equilibria.

1. Candidate Entry. Each group simultaneously decides whether or not to contest the elections. There is a cost $\delta > 0$ of contesting the elections. Let $e_i = 1(0)$ indicate that group $i$ contests (does not contest) the elections. Hence, given an entry profile $e = (e_1, e_2, \cdots, e_N)$, the set of parties contesting an election can be denoted by $\mathcal{C}(e) = \{i \in \mathcal{N} : e_i = 1\}$.
2. Elections. Citizens simultaneously vote over the set of contesting parties $\mathcal{C}(\neq \emptyset)$. We assume throughout this model that voting is costless, and each citizen votes for the party that is closest to his ideal policy. Upon elections, each party receives seats in the parliament in proportion to its relative vote share. In reality, there are minimum floor requirements and integer constraints that must be taken into account. In our model, we abstract from these considerations.
3. Government Formation. Let $(C; S_i; \{x_i\})$ denote a parliament comprised of $C(> 0)$ parties, where $S_i$ denotes party $i$'s seat share and $x_i$ its ideal point. The process of government formation is comprised of three stages: formateur selection, proto-coalition formation and the vote of confidence.
    (a) Formatuer Selection. If there is a party $k$ such that $S_k > \frac{1}{2}$, then party $k$ is asked to be the formateur. If there is a hung parliament, i.e., if $S_i \leq \frac{1}{2}$ for all $i \in \mathcal{C}$, then each party is asked to become the formateur with probability $S_i$. The formateur selection process described here (variously called proportional selection or random recognition) seems to fit the data well (see Diermeier and Merlo (2001)).
    (b) Proto-Coalition Selection. The formateur asks any subset of parties in the legislature, $D$, to form a government. $D$ is called the proto-coalition. All the members of the proto-coalition musst simultaneously decide whether or not to accept the offer. If the offer is unanimously accepted, then $D$ goes on to seek the vote of confidence; otherwise, a caretaker government is instituted.
    (c) Vote of Confidence. If a proto-coalition decides to accept the formateur's offer, it must seek the vote of confidence from the legislature. Each member of the legislature simultaneously votes to approve or to disapprove the proto-coalition. If the proto-coalition wins more than 50% of the votes, then it goes on to form a government; otherwise, a caretaker government is instituted.
4. Policy Selection. Let $D$ denote the government in office. Depending upon the outcome of the government formation stage, there could either be a single-party government, a coalition government or a caretaker government in power. There are two cases to consider.
    - Single-Party or Coalition Government. Let $\pi_i$ denote the relative seat share of party $i$ in the government. We assume that the policy chosen by such a

government is given by $\sum_{k \in \mathcal{D}} \pi_k x_k$ and each member of each party in power gets a transfer equal to $\frac{1}{\sum_{i \in \mathcal{D}} S_i} P$.

- Consensus Government. In case of a caretaker government, the policy implemented is given by $\sum_{i \in \mathcal{C}} S_i x_i$, and each member of the legislature gets a transfer $P$. In other words, a caretaker government is the same as a consensus government formed by all parties in the legislature.

We assume that if no group decides to form a party, then each citizen receives a payoff $u_{\emptyset}$.

## 4. Solving the Legislative Model

We will solve the game described in Section 3 by backwards induction (this section and the next (Section 5) are closely based on [43]). Thus, we will first solve for the coalition formation and policy-making stage for a given legislature. In the next section, we shall look at party formation and study the incentives generated by the parliamentary game for party formation. Thus, in this section, we start with a given seat share for each party. There are two stages in the legislative game, viz. government formation and policy making.

### 4.1. The Government Formation and Policy-Making Game

We assume that each party in the legislature acts as a cohesive decision making unit that tries to maximize the payoff of its representative member. Once the coalition wins the confidence (investiture) vote, policy making and division of the spoils of office is decided by bargaining among members The bargaining procedure is, of course, complicated by the fact that we have a non-constant sum game. Thus, we do not explicitly model this but simply assume that each member's strength is the weight it has in the government. Thus, they will share the rents from office in that ratio, and the implemented policy will be a seat-weighted outcome of the members' ideal points.

Let $v_i(D)$ denote the average payoff of a member of party $i$ when $D$ is the ruling coalition. If $i \notin D$, then $v_i(D) = -u(|x_i - x_D|)$, and if $i \in D$, then $v_i(D) = -u(|x_i - x_D|) + \frac{1}{s_D} P$, where $s_D$ is the 'size', i.e., the seat shares of coalition D (note that party $i$ receives $\pi_i P$ part of the power. Hence, the per party member share of power is $\frac{\pi_i P}{s_i}$. Since $\pi_i = \frac{s_i}{s_D}$, we have the per capita share to be $\frac{1}{s_D} P$). Let $v_i(\mathcal{C})$ denote the payoff of a member of party $i$ when there is a consensus government. At the vote of confidence stage, the members of party $i$ will vote for the proposed government $D$ if $v_i(D) \geq u_{\emptyset}$ (also, we assume that when indifferent, a party member votes for the proposed government). Let $A(D)$ denote the set of parties that would vote for the proposed government $D$, and let $s_{A(D)}$ denote its size. If $s_{A(D)} > \frac{1}{2}$, then $D$ forms the government. Let $W$ denote the set of proto-coalitions that will win the vote of confidence. Formally, $W \equiv \{D \in 2^{\mathcal{C}} \text{ s.t. } s_{A(D)} > \frac{1}{2}\}$. Now we come to the proto-coalition selection stage. At this stage, the formateur $k$ must choose the proto-coalition. Let $Y$ denote the set of proto-coalitions that are unanimously preferred by its constituents over the status quo. Formally, $Y \equiv \{D \in 2^{\mathcal{C}} \text{ s.t. } v_i(D) \geq v_i(\emptyset)\}$. Thus, every coalition member has a veto power in that it can decide not to be in the coalition. Hence, unanimity is required among the selected members for a coalition to be formed. Let $D_k$ denote the proto-coalition most preferred by a member of party $k$, i.e., $D_k = \arg\max_{D \in W \cap Y} v_k(D)$. For simplicity, we assume that $D_k$ is unique for each $k$ (otherwise we choose with equal probability). Thus, associated with each formateur $k$, we have an equilibrium government $D_k$. Formally, a legislative equilibrium can be defined as follows:

**Definition 1.** *A legislative equilibrium is a collection of a proto-coalition $D_1, D_2, \ldots, D_N$ such that $\forall k \in C, D_k = \arg\max_{D \in W \cap Y} v_k(D)$.*

Note that existence is not a problem as the sets $W$ and $Y$ are finite sets. Hence, $D_k$ is well defined.

### 4.2. Defining Different Coalitions

Before stating our main results on the parliamentary stage, it is useful to make precise the types of coalitions we had described in the introduction. Let $(C, S_i; \{x_i\})$ denote a parliament comprised of $C(>0)$ parties, where $S_i$ denotes party $i$'s seat share and $x_i$ its ideal point. Let $D \subset N$ denote the coalition in power, with $\pi_i$ denoting the relative seat share of party $i \in D$. Naturally, for $i \in D, \pi_i = \frac{S_i}{\sum_{k \in D} S_k}$. Some special cases of interest are

- $|D| = 1-$ a single party is in power;
- $\sum_{k \in D} S_k \leq \frac{1}{2} - D$ is a minority government;
- $\sum_{k \in D} S_k > \frac{1}{2}$ and $\exists i \in D$ such that $\sum_{k \in D \setminus i} S_k > \frac{1}{2} - D$ is a super-majority government;
- $\sum_{k \in D} S_k > \frac{1}{2}$ and for any $i \in D$, $\sum_{k \in D \setminus i} S_k \leq \frac{1}{2} - D$ is a minimal winning coalition government;
- $D = \mathcal{C}-$ a consensus government;
- Let $\mathcal{C}(\{x_i\}_{i \in D})$ denote the convex hull of the ideal points of the coalition partners. If $\exists j \notin D$ such that $x_j \in \mathcal{C}(\{x_i\}_{i \in D})$, then $D$ is a disconnected coalition. Otherwise, $D$ is a connected coalition.

## 5. Symmetric Three Party Characterization and a 'Limiting Result'

From now on, we will use the following utility function to characterize the results of the coalition formation game:

$$u(|x_i - x|) = |x_i - x|. \tag{2}$$

Consider a legislature comprised of 3 parties, 1, 2 and 3, with $x_1 = 0, x_2 = x \leq \frac{1}{2}$, and $x_3 = 1$. We shall further assume that $S_1 = S_2 = S_3 = \frac{1}{3}$. Each party has well-defined preferences denoted by a weak ordering $\succ_i$ over the set of possible coalitions. If two or more parties prefer a coalition $D$ over the status quo $\{1, 2, 3\}$, then the $D$ succeeds in forming a government. We will completely characterize the set of equilibrium coalitions.

### 5.1. Party 2 as the Formateur

First, suppose that party 2 is chosen as the formateur. It obviously prefers $\{2\}$ over any other $D$ and will succeed in forming the government if $\{2\} \succ_1 \{1, 2, 3\}$. Here, 1's payoff from $\{2\}$ in power is $-x$ while his payoff from $\{1, 2, 3\}$ is $-\frac{1+x}{3} + P$. Hence, 1 will support the coalition if

$$-x \geq -\frac{1+x}{3} + P. \tag{3}$$

which simplifies to $x \leq \frac{1}{2} - \frac{3}{2}P$. Hence, if the above condition holds, 2 will successfully propose a minority government comprised only of itself. It his condition does not hold, 2's other alternatives are $\{1, 2\}$ or $\{1, 2, 3\}$, or $\{1, 3\}$. Since $\{1, 2\} \succ_2 \{2, 3\}$, 2 will propose $\{1, 2\}$ if $\{1, 2\} \succ_2 \{1, 2, 3\}$, which is equivalent to

$$-\frac{x}{2} + \frac{P}{2} \geq x - \frac{1+x}{3} + \frac{P}{3} \tag{4}$$

which simplifies to $x \leq \frac{2}{7} + \frac{6}{7}P$. Note that 1 will always support $\{1, 2\}$ over $\{1, 2, 3\}$, and hence 2 is assured of winning the vote of confidence. If $x > \frac{2}{7} + \frac{6}{7}P$, then 2's next best alternative would be either $\{2, 3\}$ or $\{1, 2, 3\}$. In either case, the government is assured of party 3's support. Furthermore, 2 will propose $\{2, 3\}$ if

$$-\frac{1+x}{2} + 2P \geq x - \frac{1+x}{3} + P \tag{5}$$

which simplifies to $x \leq -1 + 6P$. Otherwise, 2 will prefer the status quo $\{1, 2, 3\}$.

Figure 1 summarizes the various possible coalitions in the $(P, x)$ space.

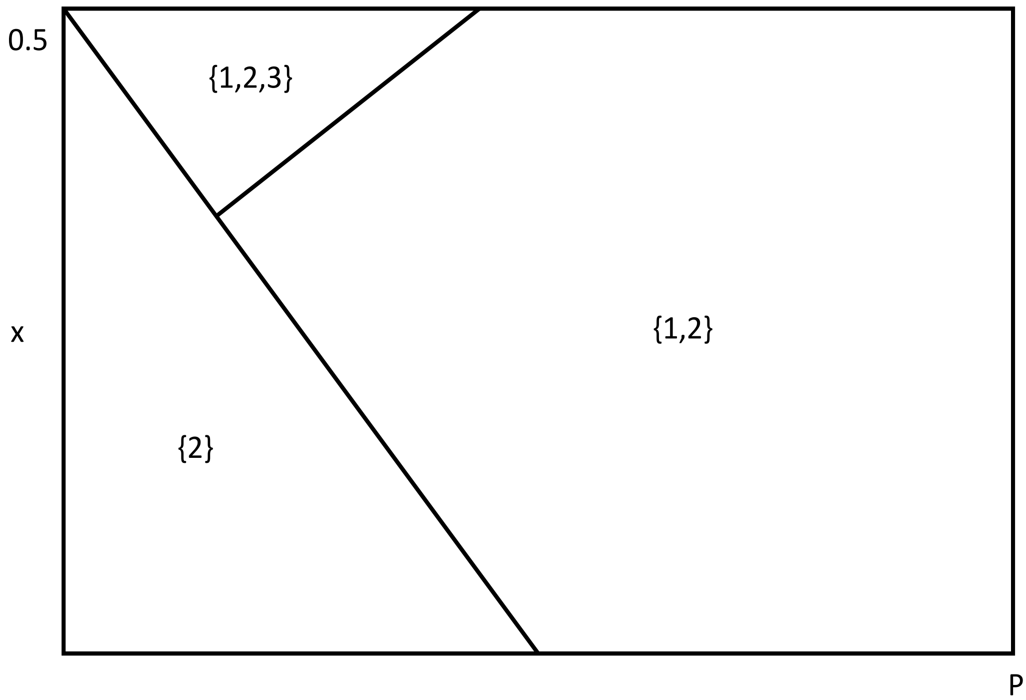


**Figure 1.** Coalitions with 2 as formateur.

### *5.2. Party 1 as the Formateur*

Let party 1 be the formateur. Its most preferred government is $\{1\}$, which it will succeed in forming if $\{1\} \succ_2 \{1,2,3\}$, which is the same as

$$-x \geq x - \frac{1+x}{3} + P \tag{6}$$

which simplifies to $x \leq \frac{1}{5} - \frac{3}{5}P$. If the above condition fails to hold, the next best feasible alternative for party 1 is $\{1,2\}$. Party 2 will accept 1's proposal to form $\{1,2\}$ if

$$-\frac{x}{2} + 2P \geq x - \frac{1+x}{3} + P, \tag{7}$$

which boils down to $x \leq \frac{2}{7} + \frac{6}{7}P$. If neither $\{2\}$ nor $\{1,2\}$ are feasible, then 1 could propose either $\{2\}, \{1,3\}$ or $\{1,2,3\}$.

Note that $\{2\}$ will always get party 2's support. Hence, 1 would propose $\{2\}$ if $\{2\} \succ_1 \{1,3\} \succ_1 \{1,2,3\}$. The former condition is equivalent to $x \leq \frac{1}{2} - 2P$, and the latter is equivalent to $x \leq \frac{1}{2} - 3P$. Similar conditions can be obtained for the range over which $\{1,3\}$ is the best feasible combination. Figure 2 shows the various equilibrium coalitions when party 1 is the formateur.

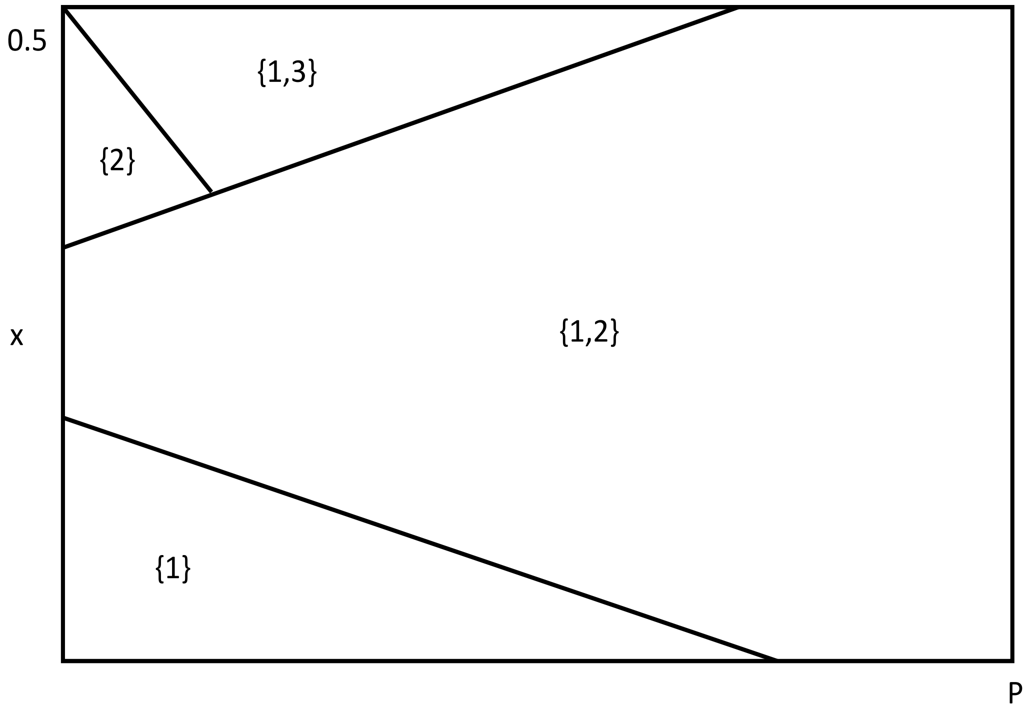


**Figure 2.** Coalitions with 1 as formateur.

### *5.3. Party 3 as the Formateur*

To study the possible coalitions when party 3 is the formateur, we do a similar exercise of deriving 3's best feasible coalitions. Figure 3 shows the various equilibrium coalitions when party 3 is the formateur.

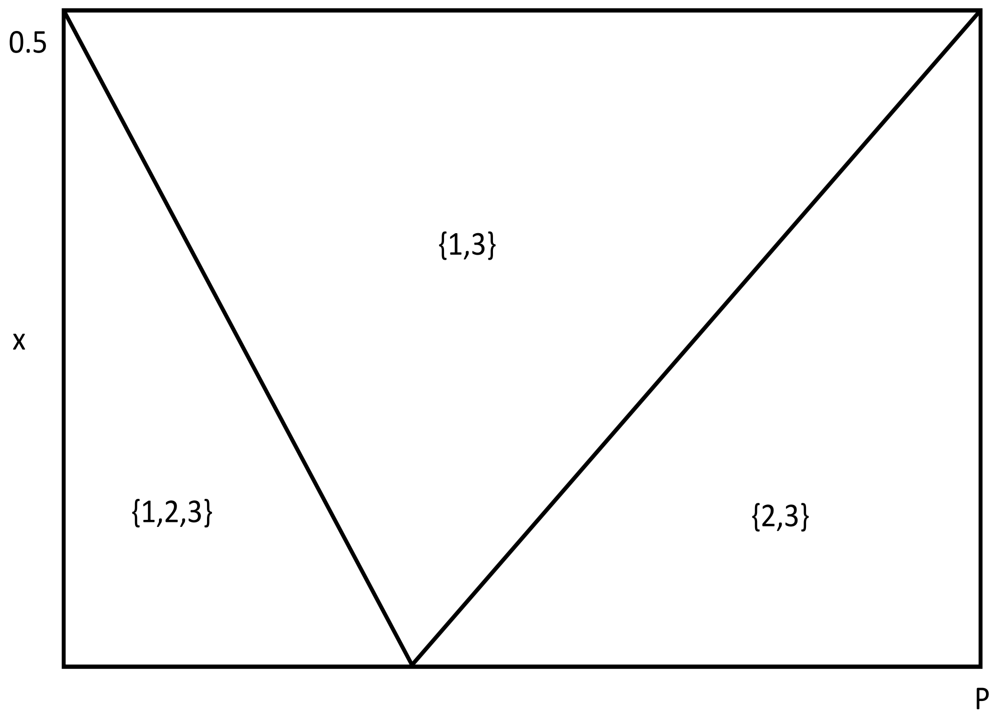


**Figure 3.** Coalitions with 3 as formateur.

### 5.4. Connected vs. Disconnected Coalitions

An important insight that the empirical work on coalitions has revealed (see Indridason, earlier cit.) is that disconnected coalitions may be more frequently seen where ideology is less important as compared to rents (or what we also interpret as special transfers as opposed to policies which affect all groups). To see when this may be true more clearly, consider two particular cases, one where there is a large party which is centrally located and two smaller parties on either side, and another where there are two ideologically similar parties with a large party further away from them. We can to fix ideas, assume, as in the symmetric case, that we have parties 1, 2, 3 with ideal points $(0; x; 1)$. $S2 \geq \max(S1; S3)$. Thus, essentially we now introduce asymmetry in party size to see how that affects coalition formation. The closer $x$ is to $\frac{1}{2}$ , the lower the value of $P$ needed to get a disconnected coalition. Again, as $S_2$ gets bigger, the chance of a disconnected coalition increases (until $S_2 = \frac{1}{2}$). This captures the intuitive phenomenon that the centrist party is left out as it is asking for too much (in terms of share of $P$). However, this is a special case of a more general result, namely that as the value of $P$ gets larger, the equilibrium coalitions are minimum winning (subject to the formateur being in the coalition). Hence, in this case the two smallest parties form a coalition and, being on either side, the coalition is disconnected. On the other hand, when they are on the same side, we can again get disconnected coalitions when $P$ is very high as the far extreme party will call on the smallest partner which may be farthest from it. In both cases, what is of further interest is to check for consistency with the entry game, which we do in the next section.

In this context, we note that the general result when the value of rents become very high is that every formateur can form a coalition that will be minimal winning. We state this formally as follows.

**Proposition 1.** *There exists a value of $P$ beyond which every formateur $i \in N$ can form a coalition, and that will be a minimum winning coalition; in particular, it will be the smallest minimum winning coalition subject to inclusion of the formateur.*

**Proof.** Consider party $i \in N$, let $M$ denote the smallest minimum winning coalition subject to inclusion of $i$, and denote by $S \geq \frac{1}{2}$ the relative size of the coalition. Denote an alternative larger coalition by $M'$ and its relative size by $S'$. Note that there exists $P$ for which $u_M + \frac{P}{S} > u_{M'} + \frac{P}{S'}$. Hence, a larger coalition is ruled out. Now we need to show that a smaller coalition will be voted down by a majority. Denote the smaller coalition by $M''$ and its relative size by $S'' < \frac{1}{2}$. It will be voted against by all members not included in $S''$ as long as $u_\emptyset + P > u''_M$. The value of $P$ which satisfies both inequalities is the one beyond which all equilibrium coalitions are minimal winning. □

## 6. Party Formation

We are now able to define the political equilibrium. We first define the entry stage equilibrium and then the political equilibrium.

**Definition 2.** *Entry-stage Equilibrium: A profile $e$ of entry decisions constitutes an equilibrium if, for all $i \in C$, $V_i(C) - \delta > V_i(C')$, where $V_i(C)$ (respectively, $V_i(C')$) is the expected utility of party $i$ from contesting (respectively, not contesting) and the set of entrants is denoted by $C$ and $C' = C - i$.*

**Definition 3.** *Political Equilibrium: A political equilibrium is a collection $\{D^*, e^*\}$ where $D^* = (D_1, D_2, \ldots, D_N)$ is a collection of equilibrium proto-coalitions of the government formation game and $e^*$ is an entry-profile such that*

1. *$\forall k \in C$, $D_k = \arg\max_{D \in W \cap Y} v_k(D)$;*
2. *$e^*$ is an equilibrium of the entry game given the proto-coalition decision functions.*

*Results*

Given these definitions we can now easily show existence.

**Proposition 2.** *A political equilibrium (possibly in mixed strategies) exists.*

**Proof.** The number of players ($1 < N < \infty$) and the strategy set is finite. Hence, the conditions for the existence of a Nash equilibrium holds. In particular, the government formation sub-game associated with each formateur also has an equilibrium as $D_k$ is well defined. The equilibrium may involve mixed strategies, as in, the groups may be played as a mixed strategy at the party formation/entry stage or at the coalition formation stage. □

Now, since the entry decision of each party is dependent on the decisions by other parties, it is not very difficult to see that we get multiple equilibria. We shall demonstrate this by giving examples of such multiplicity. However, as our next proposition shows, if the median is unique, then the median group being the only group to form a party is always an equilibrium.

**Proposition 3.** *If the median group is unique, there exists a political equilibrium in which the median group stands uncontested and implements its ideal policy in parliament.*

**Proof.** If the median group forms a party no group can get more than half the votes by standing on its own. As the median party retains its absolute majority, it still becomes the formateur and implements its ideal point. Thus, any group $i$ by launching a party incurs a net cost since its change of utility from standing is $U(x_m) - U(xm) - \delta = -\delta < 0$. Hence, no unilateral deviation is profitable. Any group by forming a party only undergoes a cost. (This is not robust in the sense that it depends on the simultaneity of the game. Note that this non-robustness is true for the citizen candidate model as well). □

It is natural at this stage to ask what (if anything) can be said about Duverger's law. Non-Duvergerian predictions for PR have been made (see the papers cited in Section 2), in particular showing that only two parties can form under PR. The assumptions are open to question but clearly it is worth investigating if strategic entry in our particular framework can give rise to non-Duvergerian predictions. An interesting point in this context made in [31] is that in a multi-district model, if the population distribution across districts is sufficiently dissimilar, Duverger's predictions are reversed. We demonstrate that even in a one district scenario, we can get more parties under plurality voting than under PR.

Suppose there are three groups with ideal points $0, x < \frac{1}{2}, 1$. We further assume that $N_i = \frac{1}{3} \forall i$. We wish to look at conditions under which the median group forming the party is the unique equilibrium. In other words, contrary to the Duvergerian prediction, we have only one party under PR. We also examine under what conditions we have another equilibrium in which all the groups will stand. To understand why the unique equilibrium could be the party with the ideal point $x$ standing, note that the group further away from $x$ (i.e., the party with ideal point 1) may wish to withdraw in order to prevent the minimum winning coalition of $0, x$ whose policy $\frac{x}{2}$ will be worse for the group than if the ideal policy of the middle group $x$ is implemented. The conditions that need to be satisfied for this is that the equiprobable chance of the three coalitions that occur when all three groups form parties must be less than the utility from $x$ being implemented with certainty. In this, we have different cases to consider. The first is when there a coalition between the parties with ideal point 0 and 1 when the party with ideal point 0 is the formateur, a minority government when the party with ideal point $x$ is the formateur, and a caretaker government when the party with ideal point 1 is the formateur.

**Proposition 4.** *Let there be three equal-sized groups, 1, 2 and 3, with the ideal points given by $0, x < \frac{1}{2}, 1$, respectively. Further assume that $|\frac{x}{2}| < |\frac{x+1}{3} - x| < |x|$. There exists a unique political equilibrium where the party with ideal point $x$ contests when the following conditions hold: (1)*

$-\frac{1}{3}|\frac{x+1}{3}-1| - \frac{1}{3}|\frac{x}{2}-1| - \frac{1}{3}|x-1| + \frac{P}{3}\delta < -|x-1|$*, and (2)* $3P - 3\delta \geq \max\{u_{\emptyset}, -x, -(1-x)\}$*, which denotes the payoff when no party runs for elections.*

**Proof.** The assumption $|\frac{x}{2}| < |\frac{x+1}{3} - x| < |x|$ implies that if all three groups have contested (and by sincere voting gained equal seat shares in the legislature), the party with ideal point 0 as the formateur will form a minimum winning coalition with the party with ideal point $x$, while the party with ideal point $x$ will form a minority coalition at low values of $P$, and the status quo will be implemented when the party with ideal point 1 is the formateur. Clearly, this is not restrictive in that in the event that the party with ideal point 0 can form a minority government, the incentive for the party with ideal point 1 to drop out is even greater. Given that, we see by condition (1) that the group with ideal point 1 prefers to drop out, which leads to the policy $x$ being implemented with certainty. Condition (2) makes sure that group 2 would always like to form a party and contest the elections. □

The intuition for this is that the further extreme party (with ideal point 1) prefers $x$ as the policy to a coalition which would lead to $\frac{x}{2}$ being the policy and hence drops out to give the middle party a majority. Thus, while several formalizations of Duverger's law relied on voters behaving strategically (see references earlier cit.), we have shown that even with sincere voting, the intuition is not very different if there is strategic behavior on the part of parties. In particular, notice that the condition for uniqueness under PR is weaker than that under plurality voting (PV) in the sense that under PV, $-\frac{1}{3}|\frac{x+1}{3}-1| - \frac{1}{3}|\frac{x}{2}-1| - \frac{1}{3}|x-1| + P - \delta < -|x-1|$ would need to hold as in the event of a tie there is a one-third probability of the party with ideal point 1 being the winner and hence appropriating all the $P$.

We now look at entry-proofness for the two asymmetric cases described in the discussion on connected vs. disconnected coalitions. Recall that we had three parties, 1, 2, and 3, with ideal points $(0, x, 1)$ and $s_2 \geq \max(s_1, s_3)$. The closer $x$ is to $\frac{1}{2}$, the lower the value of $P$ needed to get disconnected coalitions. Again, as $s_2$ gets bigger, the chances of a disconnected coalition increase (until $s_2 = 1$). We can see why at low $P$ and a reasonably moderate $\delta$, an extreme party may want to drop out.

Now consider the three parties 1, 2, 3 with ideal points $(0, x, 1)$ with $s_3 \geq \max(s_1, s_2)$. It is easy to see that with low $P$, the status quo is implemented. As an example of entry (non-proofness), consider $x = \varepsilon$ close to 0. It is easy to see that there exists $P$ such that

$$-|s_2 x + s_3 - x| + \frac{P}{3} > -|\frac{1-x}{2} - x| + \frac{P}{2}. \tag{8}$$

However, it should be noticed that this is not entry proof. This is because $-|x| > -\frac{1}{3}|s_2 x + s_3 - x| + \frac{P}{3} - \frac{1}{3}P - \delta$ for $x$ close to 0. Note of course that as P gets very large there exists a political equilibrium in which all groups contest. The following proposition formalizes this.

**Proposition 5.** *There exists a value of P such that all N groups contesting the election are a political equilibrium as long as there does not exist a group j such that* $N_j > \sum_{i \setminus j} N_i$.

**Proof.** We know that there exists a value of $P$ at which any formateur $i$ can form a minimal winning coalition including itself. We need to show that there exists a value at which in an equilibrium with all $N$ groups forming parties, no one will deviate. The minimum loss (assuming party $i$ is in the government only if selected to be a formateur) to a party $i$ by withdrawing is $\frac{P}{N} - \delta + Eu_N - Eu_{N_i} i$ ($Eu_N - Eu_{N_i}$ denotes difference in expected utility in terms of ideology implemented if party $i$ withdraws) which is $> 0$ for large $P$. □

## 7. Robustness: How Critical Are the Assumptions?

In this section, we study the robustness of the equilibrium to the assumptions we made about parliamentary rules (formateur selection procedure/ bargaining outcomes), voting behavior and the inability of parties to commit to positions other than their ideal points. We will not deal with the last two issues except to make brief remarks about each. However, we shall talk in some detail about two alternate ways to model legislative behavior that are common in the literature.

### *7.1. Majority Rule*

Instead of the formateur selection and coalition formation procedure, suppose Parliament operated by voting on each issue by majority rule. In a single-dimensional policy space, this would lead to the Condorcet winner. If a single issue is what matters to the groups, we get fairly sharp results for the whole political process. Consider the arbitrary $N$ groups and assume a unique median exists. In that case, we get the following results immediately.

**Proposition 6.** *If, after elections, the median of the candidates (representing different party positions) is implemented, at most, two groups put up candidates in equilibrium.*

**Proof.** First, note that more than two candidates standing cannot be an equilibrium. To see this note that if three (or more) candidates contest in the second stage there will be at least one candidate who will be non-pivotal, i.e., whose dropping out will not affect the implemented policy. Hence, it is not optimal for that candidate to contest.

We now show that there can be zero, one or two candidate equilibria. If the cost is very high, it is easy to see that no candidate will stand. To get one-candidate equilibria, first note that there exists a $c$ for which if the candidate with ideal point 0 stands, and for all $\varepsilon$, $1-\varepsilon$ will not find it worthwhile to contest and win. This implies that $u(-1+\varepsilon) > P - c$. This implies a continuum of one candidate equilibria where any candidate can stand in equilibrium. However, as $c$ decreases, the range decreases, i.e. the marginal candidate who can stand uncontested moves towards the median. Further, as $c$ decreases, we can get two-candidate equilibria symmetrically around the median. (We have assumed that the group sizes are the same for convenience and they are at the same distance from their neighbors; hence, two-candidate equilibria are possible. Otherwise, as we have shown before, we need to introduce voter uncertainty to get two-candidate equilibria). □

Notice that this contrasts with Duverger's hypothesis that PR leads to a multiparty (more than two) system. Moreover, this range around which symmetric two-candidate equilibria can occur also keeps shrinking.

**Proposition 7.** *As costs go to zero (and assuming the median is unique), the unique equilibrium is for the median citizen to form a party.*

**Proof.** We need to consider only one-candidate or two-candidate equilibria.

Consider a one-candidate equilibrium with a group $x_i \neq x_m$, where $x_m$ is the ideal point of the median candidate. Without any loss of generality, let $x_i < x_m$. Clearly, any $j$ such that $x_i < x_j \leq x_m$ can form a party and get more than half the votes. The net gain to group $j$ is $U(x_j) - U(x_i) + P - \delta > 0$ when $\delta \to 0$.

Now, consider two-candidate equilibria. We already know that they must be symmetric around the median. Let us denote the utility to the median group in these symmetric equilibria by $U(x_s)$. Since the post-election policy gets selected by majority rule, if the median group deviated and formed a party, it will get its ideal point in stage 2. Hence, by deviating, the median group gets $U(x_m) - U(x_s) + P - \delta > 0$ when $\delta \to 0$.

We already know that the median group being the only group to form a party is an equilibria. We have shown that no other equilibria exist. Hence, as costs go to zero, this is the unique equilibrium. □

A comparison with plurality voting is quite interesting. For different cost levels, we get one- or two-candidate equilibria, as in the citizen candidate model with plurality voting. As costs decrease, our prediction is extremely sharp under PR, unlike plurality voting, and it predicts a unique outcome. However, this seems hardly representative of how Parliament works. In particular, even if this were taken to be a way to make decisions, we run into problems if the policy space is multidimensional. Different results are obtained depending on how voting on different issues takes place.

### 7.2. Selection in Order

Another rule which is sometimes seen in formateur selection (and mandated by law in Greece) is selection in order as analyzed in [9]. We now look at ex post coalitions under the selection in order rule (which may be called the Austen Smith and Banks Protocol). Briefly, this involves a fixed order of asking parties to be the formateur, starting with the largest (in terms of vote shares), and then if the largest fails to form a government, the second largest, and so on. If all parties fail, a national government is formed, and the policy implemented is a status quo policy, which is implemented by a caretaker government that enjoys no power.

We now present some results which contrast with random selection.

**Proposition 8.** *If power and entry costs are low, the unique equilibrium of the political game is for the median group to be the unique party to form.*

**Proof.** We first show that if power is 'low' in the parliamentary game, only the median party will be able to command a majority support. This is because any coalition will have an implemented policy $x_j \neq x_m$, where $x_m$ is the median party's ideal point. Hence, a majority of members prefer $x_m$ to $x_j$. Hence, the optimal coalition when the median party proposes is for it to propose a coalition consisting only of itself, which will be accepted. Therefore, the unique equilibrium of the parliamentary game is for $x_m$ to get implemented. Clearly, if this is the outcome in the legislature, no other group will launch a party in the party formation stage. □

**Proposition 9.** *If the status quo policy $x_\emptyset$ is implemented by a caretaker government that enjoys no power, that government will not form in equilibrium.*

**Proof.** We are required to show that at least one party can form a successful coalition when it is the formateur. Notice that as $x_\emptyset$ lies between $x_1$ and $x_n$, a coalition of the median party is preferred by a majority to the status quo. Therefore, there exists a feasible coalition which dominates the status quo. □

We note that this result contrasts with that under proportional selection. In fact, while caretaker governments are not unheard of, it is usually the case that even though governments may not form at the first attempt it is almost always the case that some coalition comes to power. The one-shot version of the random recognition protocol by cutting off the game in one stage does not allow for any other party to get a chance to propose, leading to this 'extreme' situation. A further insight that we get is the following.

**Corollary 1.** *An extreme party cannot form a minority government.*

**Proof.** To see this, notice that the middle party's minority government is preferred by a majority of members. Thus, there exists at least one winning majority coalition. Hence, parties commanding a majority of seats will not accept the proposal of a minority government by an extreme party. □

A couple of remarks at this point are in order.

**Remark 1.** *Minimal winning, minority and surplus governments are possible in equilibrium. Moreover, the coalitions may be connected or disconnected.*

The trade-offs involved are similar to the proportional selection model.

**Remark 2.** *The first party may not be able to form a coalition. Hence, delays may occur in equilibrium. However, it is worth noting that for every equilibrium involving delay, there is an equilibrium without delay that leads to the same government.*

Notice that the formateur may not be able to form a government including itself, and hence, the offer it makes to other parties to join a coalition with it will be turned down. However, an equivalent outcome can be achieved by the formateur proposing a coalition excluding itself, which lies in $W \cap Y$. Some points of difference are worth noting.

First, in the random recognition protocol, we may get caretaker governments as well as minority governments. Selection in order never leads to a caretaker government in equilibrium. Moreover, only a median party can form a minority government, and that, too, only when power is very low. At the empirical level, while selection in order is not borne out, it is worth investigating if the predictions of the one-period random recognition model used in papers by [5,30], which we have adopted here as well, captures important features of the data. Clearly, finite periods of these protocols change the results, but it is still not clear what institutional details correspond to this random recognition protocol. (For a critique of the random recognition protocol, see [44]). Thus, when the largest party is not selected, we need to see if this is because a party other than the largest has indicated that it has the support of other parties, which would enable it to form a government. Another important thing to look at is how well the 'random recognition' model fits the data after accounting for an incumbency bias, i.e., where the last party in power is first asked to form the government. The 'selection in order' protocol is something that can be observed and legislated on (as in Greece). However, there is certainly a lot to be said for this 'random selection' procedure in terms of capturing the inherent uncertainty that is associated with the political environment in government formation in most countries. Moreover, this random selection model of BDM we have analyzed under modified assumptions leads to fairly interesting results.

Clearly, there are issues that are important in the political process which we have left out. For instance, we do not consider the issue of strategic voting. As long as representation per se is important, there are fewer reasons for voters to behave strategically under PR than under PV. Further work is certainly needed in this area. We have embedded a citizen candidate model in the institutional framework of coalition government formation. Thus, we do not allow parties to credibly commit to positions other than their own. Given that parties may have access to a credible commitment device (often repeated play ensures that; see [25]), it would be interesting to see if this would lead to more divergence or more convergence of party policies. This remains a fascinating area of future research.

## 8. Empirical Relevance and Concluding Remarks

We have presented a model of parliamentary democracy under PR which predicts political coalition formation as a function of party size and the relative importance of power to ideology. Moreover, by endogenizing the political entry stage, we have shown how our legislature is consistent with a party formation game under the assumption of sincere voting. Our coalition- and policy-making stages, in particular, give rise to certain predictions that contrast with those existing in the literature. In particular, two limiting cases arise, one when parties care only for ideology and another where the rents of office become very large. We find that in the limit, Riker's size principle, discussed in [15], does apply subject to inclusion of the formateur, and coalitions are minimal winning (though not minimum size) when the vale of rents from office become very big. However, because we explicitly consider party size, 'disconnected coalitions' can occur even with parties that are purely driven by ideology. Thus, when parties care only for ideology, they may leave out an ideologically close partner because a large party can tilt the policy too close towards its ideal point due to its increased bargaining strength.

From a theoretical angle, this paper shows that there can be strategic exit under PR as well PV and that a crucial incentive under all electoral systems is how the presence or absence of a party can affect the parliamentary negotiations over the final policy. We have deliberately compared our model with a stylized model of PV to show how there are conditions under which the incentives for entry are in fact higher under PV, contrary to Duverger's predictions. All of these assume that parties care for ideology. If they do not, we do see high political entry under PR. Our paper shows how the institutional details of parliamentary negotiation as well as the relative weight parties place on ideology compared to rents from office affect both the types of governments formed and political entry.

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