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Abstract

Purpose – The purpose of this paper is to develop data envelopment analysis (DEA) models and algorithms for efficiency improvement when the inputs and output weights are restricted and there is fixed availability of inputs in the system.

Design/methodology/approach – Limitation on availability of inputs is represented in the form of constant sum of inputs (CSOI) constraint. The amount of excess input of an inefficient decision-making unit (DMU) is redistributed among other DMUs in such a way so that there is no reduction in their efficiency. DEA models have been developed to design the optimum strategy to reallocate the excess input.

Findings – The authors have developed the method for reallocating the excess input among DMUs while under CSOI constraint and parameter weight restrictions. It has been shown that in this work to improve the efficiency of an inefficient DMU one needs the cooperation of selected few DMUs. The working of the models and results have been shown through a case study on carbon dioxide emissions of 32 countries.

Research limitations/implications – The limitation of the study is that only one DMU can expect to benefit from the application of these methods at any given time.

Practical implications – Results of the paper are useful in situations when decision maker is exploring the possibility of transferring the excess resources from underperforming DMUs to the other DMUs to improve the performance.

Originality/value – This strategy of reallocation of excess input will be very useful in situations when decision maker is exploring the possibility of transferring the excess resources from underperforming DMUs to the other DMUs to improve the performance. Unlike the existing works on efficiency improvement under CSOI, this work seeks to address the issue of efficiency improvement when the input/output parameter weights are also restricted.

Keywords Performance measurement, Data envelopment analysis, Efficiency, Performance evaluation, DEA, Input/output analysis, Assurance region

Paper type Research paper

1. Introduction

Data envelopment analysis (DEA) is a non-parametric methodology for frontier estimation when the firms or decision-making units (DMUs) are producing multiple outputs by consuming multiple inputs. DEA is used to calculate the relative efficiency of DMUs by comparing the individual DMU with the best practice in the group/sample. DEA was first introduced by Charnes *et al.* (1978) as an extension to the Farrell's (1957) work on productive efficiency. Main advantage of DEA over

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other efficiency measurement techniques is that neither we need a functional form of the production function nor the information about prior weight selection for input and output parameters. Efficiency measurement is very important and critical for the business managers and decision makers for any future course of action. DEA has been used in number of practical problems. Examples include, production process (Amirteimoori and Emrouznejad, 2012), banking (Avkiran, 2009; Yang *et al.*, 2010), health care (Araujo *et al.*, 2014; Karagiannis and Velentzas, 2010; Janadaghi *et al.*, 2010), sports (Singh and Adhikari, 2015; Ruiz *et al.*, 2013; Singh, 2011). For more state of the art applications of DEA in different areas, a suvey paper by Cook and Seiford (2009) may be referred. DEA makes possible the estimation of the production frontier under constant returns to scale (CRS) using the Charnes-Cooper-Rhodes (CCR) models (Charnes *et al.*, 1978) as well as under variable returns to scale (VRS) using Banker-Charnes-Cooper (BCC) models (Banker *et al.*, 1984).

One of the main assumption in basic DEA models is that a DMU can freely change its inputs used and ouputs produced without having any impact on the efficiency of the remaining DMUs. In addition to this, a DMU can assign any non-negative weights to its input and ouput parameters, i.e. inputs and outputs are freely substitutable. In this paper, we have tackled a scenario where none of these assumptions hold. Equivalently, we have handled a situation of restrictive input/output parameter weights and fixed inputs availability in the system which is represented in the form of constant sum of inputs (CSOI) constraint.

CSOI constraint occurs in many real life applications, examples include, labor and office space available to different units of the same organization, and the total amount of CO_2 emissions allowed to the various countries under the Kyoto Protocol. In the case of CO_2 emissions, we have illustrated the results with the help of the data for 32 countries. Under the Kyoto Protocols, the sum total of greenhouse gas emissions allowed to member nations is a fixed quantity. If a nation exceeds its allotted quota, it is allowed to make it up by purchasing carbon credits from more efficient members. As such, this is a problem with international implications, and it is also a perfect case of the CSOI constraint. In this study, we have developed DEA models that will help a country improve its efficiency (with weight restrictions) by trading away excess emissions, without reducing the efficiency of other countries. The last part is particularly important. By identifying those countries that can accept the CO_2 emissions without losing efficiency, we can develop a strategy for efficiency improvement that will not have difficulty finding cooperation to implement it. The method designed in this paper is a two-step process. In the first step, we identify the limit to which countries can increase emissions without losing efficiency. In the second step, we find the minimum amount of reallocaiton necessary for the target country to improve its efficiency as much as possible.

The remainder of the paper proceeds as follows. Section 2 contains the literature survey. In Section 3, we present notations, methods, and algorithms developed. The theoretical results have been applied to a case study in Seciton 4. In Section 5, we have presented a summary of the work done in this paper.

2. Literature review

In the literature, some researchers have focussed their attention on CSOI constraint. Work on this topic include Beasley's (2003) fixed-cost allocation model and Guedes de Avellar *et al.*'s (2007) Spherical Frontier Model of fixed-cost allocation.

Cook and Kress (1999) also formulated a fixed-cost allocation model. The limitation of these works is that they treat the inputs under CSOI as a fixed cost to be allocated to all the DMUs. As a result, using any of these methods means changing the inputs of all the DMUs involved. Such an action is often redundant if only one or two DMUs are actually interested in changing their input quantities. When only a single DMU is trying to improve efficiency under CSOI, using a fixed-cost allocation method such as those mentioned previously is a very inefficient approach. A similar situation of only fixed sum of outputs was addressed in Yang *et al.*'s (2011) work on competition strategy. Recently, Singh and Majumdar (2014) have also developed an efficiency improvement strategy under CSOI constraint.

However, both Singh and Majumdar (2014) and Yang *et al.* (2011) have developed DEA models under the assumption of no input and output weight restrictions. These strategies may not be sufficient in situations when any set of DMUs wants to incorporate the restriction on input/output parameter weights such as in Taylor *et al.*'s (1997) work on efficiency in Mexican banks. This paper seeks to address this issue that arises from the existing research, i.e., the best way for a DMU to improve its efficiency when input/output weights are restricted and DMUs are operating under CSOI constraint. Over the years, several different approaches to weight restrictions have been developed in DEA. Dyson and Thanassoulis (1988) discussed how weight restrictions are necessary to prevent unrealistic weights being used, and developed weight restrictions using regression analysis. Weights can be restricted to absolute values (Podinovski, 2004) or involve more general measures such as the cone-ratio method (Charnes *et al.*, 1989). In this paper, we have focussed on restrictions on the input/output weights through the assurance region (AR) method (Cooper *et al.*, 2007). AR wroks as a range between which the ratio of the weights of the input or output parameters must fall. It is a simple measure that is general enough to easily find practical applications (Thompson *et al.*, 1986) which is why we have chosen to build our model around it.

3. Model formulation and results

Below, we give the notations used in this paper.

θ_k: efficiency of the kth DMU.
θ_k^{AR}: efficiency of the kth DMU under AR method.
u_r: the weight assigned to the rth output.
v_j: the weight assigned to the jth input.
u_0: value representing the variable part of VRS DEA models.
x_{ij}: jth input of the ith DMU.
y_{ir}: rth output of the ith DMU.
n: the number of DMUs.
m: the number of inputs.
s: the number of outputs.
f_{kj}: the amount being reduced from the jth input under CSOI from the kth DMU.
s_{ij}: the amount being added to the jth input of the ith DMU, $i \neq k$.
ε: an infinitesimally small positive value.
l_{zj}^k: maximum amount that can be safely transferred from the jth input parameter of the kth DMU to the zth DMU.
$p_{a,b}$: the lower limit of the ratio $(v_a/v_b)(a, b = 1, \ldots, m, a \neq b)$.

$q_{a,b}$: the upper limit of the ratio $(v_a/v_b)(a, b = 1, \ldots, m, a \neq b)$.
$P_{a,b}$: the lower limit of the ratio $(u_a/u_b)(a, b = 1, \ldots, s, a \neq b)$.
$Q_{a,b}$: the upper limit of the ratio $(u_a/u_b)(a, b = 1, \ldots, s, a \neq b)$.

Consider the following input-oriented BCC model to evaluate the efficiency of the kth DMU:

$$\theta_k = \text{Max}\frac{\sum_{r=1}^{s} u_r y_{kr} + u_0}{\sum_{j=1}^{m} v_j x_{kj}}$$

subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{ir} + u_0}{\sum_{j=1}^{m} v_j x_{ij}} \leqslant 1, i = 1, \cdots, n,$$

v_j, $u_r \geqslant 0$, $j = 1, \ldots, m$, $r = 1, \ldots, s$, u_0 free in sign.

In this paper, we focus on the AR method (Cooper *et al.*, 2007) for restricting the parameter weights. In this method, the restriction is expressed as a range between which the ratio of the weights of the input parameters must fall. For example:

$$p_{a,b} \leqslant \frac{v_b}{v_a} \leqslant q_{a,b}$$

$$\Rightarrow p_{a,b} v_a \leqslant v_b \leqslant q_{a,b} v_a.$$

Addition of similar constraints for input and output weights as necessary, to the BCC model, we obtain the following BCC-AR model:

$$\theta_k^{AR} = \text{Max}\frac{\sum_{r=1}^{s} u_r y_{kr} + u_0}{\sum_{j=1}^{m} v_j x_{kj}}$$

subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{ir} + u_0}{\sum_{j=1}^{m} v_j x_{ij}} \leqslant 1, i = 1, \cdots, n,$$

$p_{a,b} v_a \leqslant v_b \leqslant q_{a,b} v_a$, a, $b = 1, \ldots, m$, $a \neq b$, $P_{a,b} u_a \leqslant u_b \leqslant Q_{a,b} u_a$, a, $b = 1, \ldots, s$, $a \neq b$, v_j, $u_r \geqslant 0$, $j = 1, \ldots, m$, $r = 1, \ldots, s$, u_0 free in sign.

Here, it can be assumed, without any loss of generality, that the first d input parameters are under CSOI constraint. Since these constraints are under CSOI, therefore, the sum of all changes must be 0. Thus, if the jth input of the kth DMU, i.e. x_{kj} is reduced by a certain amount f_{kj} then the value of the jth inputs of the other DMUs will have to be increased. Let $s_{ij}(i \neq k, i = 1, \ldots, n)$ be the amount by which the jth input of the ith$(i \neq k)$ DMU is increased, then $f_{kj} = \sum_{i=1, i \neq k}^{n} s_{ij}, f_{kj} < x_{kj}$. However, we need to ensure that the efficiency of the receiving DMUs is not adversely affected by the redistribution. To do this we determine the amount that can be transferred from the kth DMU to any other DMU $z(z \neq k)$. Let $l_{zj}^{k}, (j = 1, \ldots, d, z = 1, \ldots, n, z \neq k)$ be the amount that can be safely transferred from the jth input parameter of the kth DMU to the zth DMU. Let θ_z^{AR*} be the original BCC-AR efficiency of the zth DMU. l_{zj}^{k} can be determined using the model ($M1$), which ensures there is no efficiency reduction for the zth DMU during this input transfer:

$$(M1)\text{Max}\sum_{j=1}^{d} v_j l_{zj}^k$$

subject to :

$$\theta_z^{AR*} \leqslant \frac{\sum_{r=1}^{s} u_r y_{zr} + u_0}{\sum_{j=1}^{d} v_j\left(x_{zj} + l_{zj}^k\right) + \sum_{j=d+1}^{m} v_j x_{zj}} \leqslant 1,$$

$$\frac{\sum_{r=1}^{s} u_r y_{kr} + u_0}{\sum_{j=1}^{d} v_j\left(x_{kj} + l_{zj}^k\right) + \sum_{j=d+1}^{m} v_j x_{kj}} \leqslant 1,$$

$$\frac{\sum_{r=1}^{s} u_r y_{ir} + u_0}{\sum_{j=1}^{m} v_j x_{ij}} \leqslant 1, i \neq k, z, i = 1, \cdots, n,$$

$$l_{zj}^k \leqslant x_{kj} - \epsilon,$$

$$l_{zj}^k \geqslant 0, j = 1, \cdots, d$$

$p_{a,b}v_a \leqslant v_b \leqslant q_{a,b}v_a$, a, $b = 1, \ldots, m$, $a \neq b$, $P_{a,b}u_a \leqslant u_b \leqslant Q_{a,b}u_a$, $a,b = 1, \ldots, s$, $a \neq b$, v_j, $u_r \geqslant 0$, $j = 1, \ldots, m$, $r = 1, \ldots, s$, u_0 free in sign:

Remark 1. Model ($M1$) is a non-linear programming model, but it can be transformed into an equivalent linear programming model using the Charnes-Cooper transformation, as shown in Appendix 1.

Remark 2. In DEA, the efficiency of a DMU is calculated by its relation to the efficiency frontier (Cooper *et al.*, 2007). As long as there is no change in the position of the frontier and the input/output values of a DMU are unchanged, the efficiency of the DMU remains unchanged. This means, in model ($M1$), if the kth DMU cannot reduce its inputs enough to become efficient, it will not affect the efficiency of any other DMU. Conversely, if the kth DMU becomes efficient then there is a possibility that model ($M1$) will reduce its inputs to the point where it pushes the efficiency frontier. This will reduce the efficiency of the DMUs other than the zth DMU, which is protected from efficiency reduction by the first constraint in the model ($M1$).

Remark 3. Following from Remark 2, if the kth DMU is becoming efficient, it will then become necessary to determine the minimum amount of reduction for the kth DMU to achieve efficiency. For this amount of input reduction, the kth DMU will reach the efficiency frontier without pushing the frontier. This means that for this amount, the efficiency of other DMUs will not be reduced.

Remark 4. Model ($M1$) helps us to know the amount of input that can be transferred from the kth DMU to any other zth DMU, but it does not tell us the required amount of input transfer when multiple DMUs are involved. Thus, we need to apply model ($M1$) repeatedly while constantly updating the input values of the DMUs, in order to determine the maximum transfer amounts across several DMUs.

Next, we present an algorithm to achieve the objectives set through remarks 2-4.

Algorithm

Step 1. Set the value of variables $l_{ij}^{k*} = 0(i \neq k, i = 1, \ldots, n, j = 1, \ldots, d)$ and $f_{kj}^{*} = 0(j = 1, \ldots, d)$. Let $x_{ij}^{*} = x_{ij}(i = 1, \ldots, n, j = 1, \ldots, d)$.

Step 2. Apply model $M1$ for the zth DMU ($1 \leqslant z \leqslant n, z \neq k$), where the zth DMU has the highest original AR efficiency θ_z^{AR*}. Note the resulting values of $l_{zj}^{k}, (j = 1, \ldots, d)$. Update the values $x_{zj} = x_{zj} + l_{zj}^{k}, (j = 1\ldots, d)$ and $x_{kj} = x_{kj} - l_{zj}^{k}, (j = 1, \ldots, d)$. Set $l_{zj}^{k*} = l_{zj}^{k*} + l_{zj}^{k}, (j = 1, \ldots, d)$.

Step 3. Check the new DEA-AR efficiency of the kth DMU. If $\theta_k^{AR} = 1$ then go to step 7, otherwise step 4.

Step 4. Repeat steps 2 to 3 for the rest of the DMUs except the kth DMU, going in descending order of θ_z^{AR*}. Then go to step 5.

Step 5. If all $l_{ij}^{k} = 0, (i = 1, \ldots, n, i \neq k, j = 1, \ldots, d)$ then it means no further improvement is possible to the kth DMU, go to step 6. Otherwise, repeat steps 2 to 4.

Step 6. Set $f_{kj}^{*} = \sum_{i=1, i \neq k}^{n} l_{ij}^{k*} (j = 1\ldots, d)$. The final input values of the DMUs are $x_{ij} = x_{ij}^{*} + l_{ij}^{k*}$ and $x_{kj} = x_{kj}^{*} - f_{kj}^{*} (i = 1, \ldots, n, i \neq k, j = 1, \ldots, d)$. Recalculate the final efficiency scores.

Step 7. Since the algorithm has reached this step, it means that it is possible for the kth DMU to become efficient. We now determine the minimum necessary reduction to make it efficient, so it reaches the efficiency frontier without pushing it. To do this, we first reset the inputs to their original values, $x_{ij} = x_{ij}^{*}(i = 1, \ldots, n, j = 1, \ldots, d)$. Then, taking note of the input transfer limits l_{ij}^{k*} $(i = 1, \ldots, n, i \neq k, j = 1, \ldots, d)$, we apply the model ($M2$).

$$(M2)\text{Min} \sum_{j=1}^{d} v_j f_{kj}$$

subject to:

$$\frac{\sum_{r=1}^{s} u_r y_{kr} + u_0}{\sum_{j=1}^{d} v_j (x_{kj} - f_{kj}) + \sum_{j=d+1}^{m} v_j x_{kj}} = 1$$

$$\frac{\sum_{r=1}^{s} u_r y_{ir} + u_0}{\sum_{j=1}^{d} v_j (x_{ij} + s_{ij}) + \sum_{j=d+1}^{m} v_j x_{ij}} \leqslant 1, i \neq k, i = 1, \cdots, n,$$

$$f_{kj} = \sum_{i=1, i \neq k}^{n} s_{ij},$$

$f_{kj}, s_{ij} \geqslant 0, j = 1, \ldots, d, i = 1, \ldots, n, s_{ij} \leqslant l_{ij}^{k*}, j = 1, \ldots, d, i = 1, \ldots, n, i \neq k, p_{a,b} v_a \leqslant v_b \leqslant q_{a,b} v_a$, $a, b = 1, \ldots, m$, $a \neq b$, $P_{a,b} u_a \leqslant u_b \leqslant Q_{a,b} u_a$, $a, b = 1, \ldots, s$, $a \neq b$, v_j, $u_r \geqslant 0$, $j = 1, \ldots, m$, $r = 1, \ldots, s$, u_0 free in sign.

Step 8. Final input values generated by model ($M2$) represent the input values of the DMUs at which the observed DMU k will achieve efficiency without reducing the efficiency of the other DMUs. Thus, we have reached our objective. End of algorithm:

Remark 5. Model ($M2$) is a non-linear programming model. It can be transformed into an equivalent linear programming model (see Appendix 2).

Remark 6. All the models in this work are designed for DMUs operating under VRS. If the DMUs are under CRS then the models may be modified for CRS by setting $u_0 = 0$.

4. Case study of CO_2 emissions of countries in 2012

Since the advent of the Kyoto Protocol, the total amount of greenhouse gas emissions (measured in tons of carbon dioxide) that is allowed to participant countries has become a fixed quantity. Under the protocol, a country may reduce its carbon footprint by purchasing carbon credits from less polluting countries. While carbon dioxide emissions are a product of industrial processes, since it is an undesirable output, it may be treated as an input for the purposes of efficiency calculation (Gomes and Lins, 2008). The case study is carried out on data from 32 countries with the highest carbon emissions. The data is divided into two inputs: estimated carbon dioxide emissions for 2012 (Netherlands Environment Assessment Agency, 2011), and population (Wikipedia, 2013). The output is the gross domestic product (GDP), adjusted for purchasing power, as estimated by the International Monetary Fund (2013).

To calculate the AR efficiency of the 32 countries, we must first define the limits of the AR. Cooper *et al.* (2007) identify one method of defining the bounds of the AR, as the ratio of the costs of the various parameters. Thus, in this case, the bounds of the ratio v_2/v_1 is (Min(GDP per million/CO_2 cost per kton), Max(GDP per million/CO_2 cost per kton)).

From the data provided in Table I, the highest GDP per million belongs to USA at 51.221.

Country	Input 1 CO_2 emission (kton)	Input 2 Population (millions)	Output 1 GDP (PPP) ($ billion)	VRS AR Eff.
China	9,700,000	1,361.24	12,261	0.397
USA	5,420,000	317.13	16,244	1
India	1,970,000	1,236.84	4,716	0.516
Russian Federation	1,830,000	143.6	2,486	0.382
Japan	1,240,000	127.29	4,575	0.962
Germany	810,000	80.55	3,167	0.958
South Korea	610,000	50.22	1,622	0.721
Canada	560,000	35.16	1,446	0.846
Indonesia	490,000	237.64	1,212	0.402
UK	470,000	63.71	2,312	0.977
Saudi Arabia	460,000	30	741	0.531
Brazil	450,000	201.03	2,330	0.775
Mexico	450,000	118.4	1,758	0.633
Australia	430,000	23.26	961	0.833
Iran	410,000	77.08	988	0.467
Italy	410,000	59.83	1,813	0.83
South Africa	360,000	52.98	579	0.437
France	360,000	65.81	2,252	1
Poland	350,000	38.5	802	0.505
Ukraine	320,000	45.46	335	0.492
Malaysia	310,429.33	29.79	492	0.521
Spain	300,000	46.7	1,407	0.835
Turkey	278,866.33	75.63	1,125	0.727
Taiwan	270,000	23.36	902	0.867
Thailand	230,000	65.93	646	0.646
Kazakhstan	222,990.58	17.1	232	0.74
Egypt	208,864.56	83.66	538	0.681
Argentina	195,212.22	40.12	747	0.811
Venezuela	178,217.22	29.28	402	0.876
Pakistan	174,912.11	184.88	515	0.651
United Arab Emirates	170,376.43	8.26	271	1
The Netherlands	160,000	16.81	710	1

Table I.
Output/input and data for 32 countries

The lowest GDP per million population is Pakistan at 2.785.

The highest GDP per kton CO_2 is France's 0.00626.

The lowest GDP per kton CO_2 is Kazakhstan's 0.00104.

Thus, the AR constraint for this case can be written as:

$$(2.785/0.00626)(v_1) \leqslant v_2 \leqslant (51.221/0.00104)(v_1) = 444.89(v_1) \leqslant v_2 \leqslant 49250(v_1).$$

Using the above limits, we calculate the AR efficiency under VRS for all 32 countries. The inputs, output, and AR efficiency for the countries is tabulated in Table I.

Our objective is to determine how much of its carbon emissions a country (e.g. India here) needs to transfer to other countries via purchasing carbon credits, in order to achieve efficiency. Applying Algorithm 1, we have first used model (*M*1) to transfer input from India to USA, since USA is among the most efficient countries. After applying model (*M*1), we found the algorithm skips to step 7, because step 3 has detected that India has achieved efficiency. This means that USA has enough slack in its system that India can purchase all its required carbon credits from USA. At the beginning of step 7, the limit for input transfer $l_{21}^{3^*}$ =1,970,000. Inputting this limit into model (*M*2), we can now calculate the minimum necessary CO_2 transfer for India to become efficient.

According to model (*M*2), India needs to trade 1,158,105 kton of CO_2 to the USA in order to become efficient for the year 2012. After making this change and recalculating the efficiency for all countries, we find that India now has efficiency 1, and none of the other countries show any reduction in efficiency. Thus, we have achieved our objective of improving our target country's efficiency, without adversely affecting the efficiency of other countries, even when operating under AR restrictions. The final results and new efficiency scores are all shown in Table II.

5. Conclusion and future scope

In this paper, we have proposed DEA models for efficiency improvement of an inefficient observed DMU operating under input/output weight restrictions. We have also considered fixed input scenario by way of CSOI constraint. Based on the models formulated, algorithms have been designed to aid the decision maker in efficiency improvement through redistribution of excess input among other DMUs. The excess amount of input is reallocated to other DMUs in such a way so that there is no reduction in the efficiencies of the takers. Advantage of our work over the existing ones on CSOI is that we focus on a single DMU rather than treating CSOI as a fixed-cost allocation across all DMUs. Furthermore, existing works in both CSOI and constant sum of outputs do not address the issue of weight restrictions. In the case study, we demonstrate how the best solution for India to reduce its excess CO_2 emissions is trade them to the USA over any other country.

The managerial implications of this is significant. As demonstrated in the case study, the methods developed allow us to find ways of improving the efficiency of DMUs under situations where the total resources available is a fixed quantity, without requiring cooperation from a large number of other DMUs. The algorithm in this paper is specifically made to narrow down the list of targets for excess input transfer. A DMU can improve its efficiency by gaining the cooperation of a few select DMUs. From a managerial perspective, this method greatly cuts down on the amount of effort needed to coordinate with other DMUs. Furthermore, since this method is designed to improve one DMU's efficiency while ensuring that none of the other DMUs are adversely affected, it means a manager should have an easier time gaining the cooperation of the few DMUs to whom he needs to transfer his excess input.

Table II.
New CO_2 emissions and VRS-AR efficiency

Country	Old CO_2 Emissions (kton)	Old VRS-AR Eff.	New CO_2 Emissions (kton)	New VRS-AR Eff.
China	9,700,000	0.397	9,700,000	0.472
USA	5,420,000	1	6,578,105.255	1
India	1,970,000	0.516	811,894.7452	1
Russian Federation	1,830,000	0.382	1,830,000	0.404
Japan	1,240,000	0.962	1,240,000	1
Germany	810,000	0.958	810,000	0.998
South Korea	610,000	0.721	610,000	0.753
Canada	560,000	0.846	560,000	0.87
Indonesia	490,000	0.402	490,000	0.402
UK	470,000	0.977	470,000	0.994
Saudi Arabia	460,000	0.531	460,000	0.532
Brazil	450,000	0.775	450,000	0.778
Mexico	450,000	0.633	450,000	0.633
Australia	430,000	0.833	430,000	0.845
Iran	410,000	0.467	410,000	0.467
Italy	410,000	0.83	410,000	0.837
South Africa	360,000	0.437	360,000	0.437
France	360,000	1	360,000	1
Poland	350,000	0.505	350,000	0.506
Ukraine	320,000	0.492	320,000	0.492
Malaysia	310,429.33	0.521	310,429.33	0.521
Spain	300,000	0.835	300,000	0.835
Turkey	278,866.33	0.727	278,866.33	0.727
Taiwan	270,000	0.867	270,000	0.881
Thailand	230,000	0.646	230,000	0.646
Kazakhstan	222,990.58	0.74	222,990.58	0.74
Egypt	208,864.56	0.681	208,864.56	0.681
Argentina	195,212.22	0.811	195,212.22	0.811
Venezuela	178,217.22	0.876	178,217.22	0.876
Pakistan	174,912.11	0.651	174,912.11	0.651
United Arab Emirates	170,376.43	1	170,376.43	1
The Netherlands	160,000	1	160,000	1

There can be some possible issues that may be taken as direction for future research. While this paper focusses on the AR weight restrictions, the method developed here can be easily extended to other forms of restrictions. There are many other restrictions that may be seen in real-world scenarios such as cone-ratio weight restrictions, cost restrictions, environmental constraints, etc. Finding ways to combine our method with some of these constraints is one possible direction. Another possible research goal could be to solve the biggest limitation of this method – only one DMU can expect to benefit from the application of these methods at any given time. Finding a way to extend this method across several DMUs simultaneously will greatly improve the applicability of this research.

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Appendix 1

In model ($M1$), let $\sum_{r=1}^{s} u_r y_{kr} = R, Rv_j = \varphi_j, Ru_r = \mu_r, Rv_j l_{zj}^k = \tau_j$ and $Ru_0 = \mu_0$. Since R is a positive number (as by constraints u_r and y_{kr} are both positive), both numerator and denominator of a fraction can be multiplied by R while maintaining equivalency. Multiplying both numerator and denominator by R ($M1$) can be re-written as:

$$\text{Max} \sum_{j=1}^{d} (\tau_j / R)$$

subject to:

$$\theta_z^{AR*} \left(\sum_{j=1}^{m} \varphi_j x_{zj} + \sum_{j=1}^{d} \tau_j \right) \leqslant \sum_{r=1}^{s} \mu_r y_{zr} + \mu_0,$$

$$\sum_{r=1}^{s} \mu_r y_{zr} + \mu_0 \leqslant \sum_{j=1}^{m} \varphi_j x_{zj} + \sum_{j=1}^{d} \tau_j,$$

$$\sum_{r=1}^{s} \mu_r y_{kr} + \mu_0 \leqslant \sum_{j=1}^{m} \varphi_j x_{kj} - \sum_{j=1}^{d} \tau_j,$$

$$\sum_{r=1}^{s} \mu_r y_{ir} + \mu_0 \leqslant \sum_{j=1}^{m} \varphi_j x_{ij}, i = 1, \ldots, n, i \neq k, z,$$

$\tau_j \leqslant \varphi_j x_{kj}$, $\tau_j \geqslant 0, j = 1, \ldots, d$, $p_{a,b}\varphi_a \leqslant \varphi_b \leqslant q_{a,b}\varphi_b$, $P_{a,b}\mu_a \leqslant \mu_b \leqslant Q_{a,b}\mu_a$, $a, b = 1, \ldots, s$, $a \neq b$, φ_j, $\mu_r \geqslant 0$, $j = 1, \ldots, m$, $r = 1, \ldots, s$, μ_0 free in sign.

In the transformed model, any value of R will not change the optimal solution. By setting $R = 1$, we get a linear programming model, with decision variables τ_j, μ_r, φ_j, μ_0. Solving the model for these variables, we can calculate $l_{zj}^k = \tau_j / Rv_j = \tau_j / \varphi_j$.

Appendix 2

In model (*M2*), let $\sum_{r=1}^{s} u_r y_{kr} = R, Rv_j = \varphi_j, Ru_r = \mu_r, Rv_j s_{ij} = \tau_{ij}, v_j f_{kj} = \sum_{i=1, i \neq k}^{n} \tau_{ij}$ and $Ru_0 = \mu_0$. Since R is a positive number, both numerator and denominator of a fraction can be multiplied by R while maintaining equivalency, and (*M2*) can be re-written as:

$$\text{Min} \sum_{j=1}^{d} \sum_{i=1, i \neq k}^{n} (\tau_{ij}/R)$$

subject to :

$$\sum_{j=1}^{m} \varphi_j x_{kj} - \sum_{j=1}^{d} \sum_{i=1, i \neq k}^{n} \tau_i j = 1,$$

$$\sum_{r=1}^{s} \mu_r y_{kr} + \mu_0 = 1,$$

$$\sum_{r=1}^{s} \mu_r y_{ir} + \mu_0 \leqslant \sum_{j=1}^{m} \varphi_j x_{ij} + \sum_{j=1}^{m} \tau_{ij}, i = 1, \ldots, n, i \neq k,$$

$\tau_{ij} \leqslant \varphi_j l_{ij}^{k*}, j = 1, \ldots, m, i = 1, \ldots, n, i \neq k,\ \tau_{ij} \geqslant 0,\ j = 1, \ldots, m,\ i = 1, \ldots, n,\ i \neq k,\ p_{a,b}\varphi_a \leqslant \varphi_b \leqslant q_{a,b}\varphi_b,\ P_{a,b}\mu_a \leqslant \mu_b \leqslant Q_{a,b}\mu_a,\ a, b = 1, \ldots, s,\ a \neq b,\ \varphi_j, \mu_r \geqslant 0, j = 1, \ldots, m, r = 1, \ldots, s,\ \mu_0$ free in sign.

In the transformed model, any value of R will not change the optimal solution. By setting $R = 1$, we get a linear programming model, with decision variables $\tau_{ij}, \mu_r, \varphi_j, \mu_0$. Solving the model for these variables, we can calculate $s_{ij} = (\tau_{ij}/Rv_j) = (\tau_{ij}/\varphi_j)$.

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