General connected and reconnected fields in plasmas

Swadesh M. Mahajan, and Felipe A. Asenjo

Citation: Physics of Plasmas **25**, 022116 (2018);
View online: https://doi.org/10.1063/1.5021492
View Table of Contents: http://aip.scitation.org/toc/php/25/2
Published by the American Institute of Physics

General connected and reconnected fields in plasmas

Swadesh M. Mahajan[1,2,a)] and Felipe A. Asenjo[3,b)]
[1]*Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712, USA*
[2]*Department of Physics, Shiv Nadar University, Lucknow, UP 201314, India*
[3]*Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Santiago 7941169, Chile*

(Received 4 January 2018; accepted 30 January 2018; published online 15 February 2018)

For plasma dynamics, more encompassing than the magnetohydrodynamical (MHD) approximation, the foundational concepts of "magnetic reconnection" may require deep revisions because, in the larger dynamics, magnetic field is no longer connected to the fluid lines; it is replaced by more general fields (one for each plasma specie) that are weighted combination of the electromagnetic and the thermal-vortical fields. We study the two-fluid plasma dynamics plasma expressed in two different sets of variables: the two-fluid (2F) description in terms of individual fluid velocities, and the one-fluid (1F) variables comprising the plasma bulk motion and plasma current. In the 2F description, a Connection Theorem is readily established; we show that, for each specie, there exists a Generalized (Magnetofluid/Electro-Vortic) field that is frozen-in the fluid and consequently remains, forever, connected to the flow. This field is an expression of the unification of the electromagnetic, and fluid forces (kinematic and thermal) for each specie. Since the magnetic field, by itself, is not connected in the first place, its reconnection is never forbidden and does not require any external agency (like resistivity). In fact, a magnetic field reconnection (local destruction) must be interpreted simply as a consequence of the preservation of the dynamical structure of the unified field. In the 1F plasma description, however, it is shown that there is no exact physically meaningful Connection Theorem; a general and exact field does not exist, which remains connected to the bulk plasma flow. It is also shown that the helicity conservation and the existence of a Connected field follow from the same dynamical structure; the dynamics must be expressible as an ideal Ohm's law with a physical velocity. This new perspective, emerging from the analysis of the post MHD physics, must force us to reexamine the meaning as well as our understanding of magnetic reconnection.
Published by AIP Publishing. https://doi.org/10.1063/1.5021492

I. INTRODUCTION

Magnetic reconnection in a charged fluid is one of the most investigated processes in literature. In the simplest case, this energetic event consist in the diffusion and reconnection of magnetic field lines; the latter caused by plasma resistivity. The study of magnetic reconnection is of fundamental importance since it can occur at different scales and in different situations spanning laboratory, astrophysical, and general relativistic plasmas.[1,2]

For a deeper understanding of the reconnection process (the subject of an enormous number of papers), one must necessarily begin with identifying the *physical quantity* that is expected to remain *connected* during the system evolution. This is determined by a Connection Theorem (CT). The first formulation of such a CT was worked out by Newcomb[3] in 1958. It was demonstrated that in an ideal magnetohydrodynamical (MHD) plasma (obeying an ideal Ohm's law), two fluid elements connected by a magnetic field line at some given time, will remain connected for all subsequent times, i.e., the plasma moves with just the right transport velocity that preserves the magnetic connections. This conservation law for the magnetic field lines is one of the most important ideas in plasma physics; the magnetic field emerges as a topological field whose characteristic identity is preserved by the constrained dynamics. It is only when these constraints are violated, that the magnetic field lines can reconnect and may, in the process, locally annihilate the field. The most common example is when one invokes plasma resistivity that breaks the ideal Ohm's law.

Connection Theorems (CTs) (mathematical formulation of the concepts of connection and reconnection of fields) have been derived in dissipation-less systems of greater generality where physics is much more general than the original non-relativistic MHD.[4–8] Pegoraro, for instance, showed that for an ideal relativistic MHD plasmas, there is a preserved connection,[4] where the connected field is the magnetic field. Similar CT can be extended to include general relativistic effects.[6] However, for more complicated plasma systems, the proofs for CTs are not straightforward. On the other hand, there was a major transformation in the general theory when, formulating the fully covariant dynamics of hot relativistic fluids, Mahajan[9] demonstrated that the mathematical structure of the non-relativistic MHD (epitomized in the induction law with an ideal Ohm's law) was, quite strictly, maintained for the much more encompassing larger dynamics. The effective fields in the larger dynamics are a combination of the magnetic and thermal-vortical fields (see Ref. 10 for a detailed account) implying what has been called the magneto-fluid unification. In fact, each component of the plasma obeys its own ideal Ohm's law [see Eq. (35) of Ref. 9 or Eq. (39) of Ref. 10 for

a)Electronic mail: mahajan@mail.utexas.edu
b)Electronic mail: felipe.asenjo@uai.cl

explicit expressions]. Consequently, each fluid has its own frozen-in effective field. For example, in an electron–ion plasma, the total canonical vorticity (circulation) of the ion plasma

$$\mathbf{B} + \frac{m_i}{e}\nabla \times (f_i\gamma_i\mathbf{u}_i) \tag{1}$$

(with the nonrelativistic limit $\mathbf{B} + (m_i/e)\nabla \times \mathbf{u}_i$) is frozen in the ion fluid. Here, $\mathbf{B}$ is the magnetic field, m_i is the ion mass, e is the electron charge, $\mathbf{u}$ is the velocity, γ is the kinematic relativistic factor, and f, denoting thermal enthalpy, can be viewed as a "thermal γ." Besides, the exact equivalent field, the total canonical vorticity of the electron plasma, is frozen in the electron fluid

$$\mathbf{B} - \frac{m_e}{e}\nabla \times (f_e\gamma_e\mathbf{u}_i) \tag{2}$$

(with the nonrelativistic limit $\mathbf{B} - (m_e/e)\nabla \times \mathbf{u}_e$).

The basic message of the Electro–Vortic (EV, a more covariant expression for the magneto-fluid) formulation[10] is that the plasma evolution equations do not discriminate between the electromagnetic and the thermal-vortical forces (Maxwell equations, of course, do) and what is frozen-in in a non-dissipative dynamics is the magnetic part of the Electro–Vortic field, but not either of its contributing parts.

Although Ref. 9 had worked out the basic structure of the very general non-dissipative fluid dynamics of charged particles, it did not explicitly derive the pertinent Connection Theorem (CT), or explore the consequences for reconnection. The CT theorems for the larger dynamics were explicitly derived in Refs. 7 and 8 exploiting the unified antisymmetric Electro–Vortic tensor (the version in Ref. 9). Since these connection theorems identify the connected fields to be more general than the conventional magnetic field, one must be ready to revise our ideas about the reconnection process invoked often to explain the local destruction of the magnetic field.

In fact, a simple example of the new paradigm is given in Ref. 11 that studies a non-relativistic non-ideal collisionless electron–ion plasma keeping both ion and electron inertia; it was shown that magnetic reconnection is a consequence of the conservation of the canonical circulation flux $\iint[\mathbf{B} -(m_e/e)\nabla \times \mathbf{u}_e] \cdot d\mathbf{s}$. Since the total canonical vorticity cannot change its topology, the increase in the electron fluid vorticity $\nabla \times \mathbf{u}_e$ leads to the reconnection of magnetic field lines.

In this paper, we would like to advance an alternative mode of thinking. Since the concept of reconnection makes sense only for those fields that were in some way connected, it is not meaningful to talk about *magnetic reconnection* in a dynamics more encompassing than the ideal non-relativistic MHD; the only exception is the inertialess Hall MHD, where the magnetic field is still frozen in the electron fluid, and therefore, magnetic reconnection in the electron fluid is an acceptable concept. For the general dynamics, the magnetic field lines become somewhat fictitious; it is the lines of the generalized field that have fidelity because they are connected to the streamlines of one of the fluids. The concept of the generation/destruction of the magnetic field should be replaced by the generation/destruction of the "magnetic" component (EV_m) of the EV field. The changes in the strength and character of the magnetic field are reflective of the energy transformations between the Electromagnetic and the Thermal–Vortical parts as the system evolves respecting the appropriate constraints (like the generalized helicity conservation for each species of the plasma); *magnetic topology is not the dictator of changes, but it is the topology of EV_m that undergoes a change in response to, say, dissipation.* This view point was developed in detail in relation to the so-called problem of seed magnetic field generation in a dissipation-less plasma.[10] Reconnection and magnetic field generation are the two aspects of the same phenomenon, and both are forbidden by the same topological constraints in the non-relativistic MHD. For the larger, more realistic dynamics, conceptual rethinking is necessary as the magnetic field is but a part of the unified EV field.

Before proceeding further, it is important to explicitly define some of the terminologies used in the rest of this paper. The Electro-Vortic formalism, in its original form, deals, separately, with equations of motion of each fluid constituting a plasma; the dynamic variables are the four-velocities of the individual fluid in addition to the electromagnetic fields. For a standard electron-ion or electron-positron plasma, such a formulation will be referred to as a two-fluid (2F) variable system. The alternative formalism, where the two species are combined in what may be called one-fluid (1F) variables (the four-current and the mass flow four-velocity) is more familiar, primarily because of highly studied systems such as the non-relativistic MHD and Hall MHD. Amongst other things, this paper will attempt to show how the 1 and 2F formalisms may expose different aspects of the physics of the plasma.

The 2F approach of Refs. 9 and 10 has clearly demonstrated that the magnetic part of the individual Electro-Vortic field (EV_m) is frozen in the respective fluid (the plasma could have more than two constituents). The necessary and sufficient condition for constructing a connection theorem for $\mathcal{B}$, the magnetic part of the EV field, is the existence of an ideal Ohm's law of the form[10,12]

$$\mathcal{E}_k + \mathbf{u}_k \times \mathcal{B}_k = 0, \tag{3}$$

where $\mathcal{E}$ and $\mathbf{u}$ are, respectively, the electric part of the EV field and the three vector part of the four velocity pertaining to the fluid k. Thus, the lines of $\mathcal{B}_k$ (and not the magnetic field $\mathbf{B}$) are continually tied to the streamlines of its constituent "ideal" fluid, and each fluid has its own connection theorem.

One of the questions we explore in this paper is that whether any additional independent connection theorems exist, for instance, in a 1F set-up. We will still attempt to, formally, construct a CT (and the conservation of the circulation flux) for a two component (ion–electron) one-fluid plasma evolving under the most general relativistic dynamics of Refs. 9 and 10. One must, however, be prepared to encounter inherent problems with this project. When one mixes the two fluids for a 1F description, a new composited velocity must be assigned to the total fluid. Thus, an ideal Ohm's law (that may pertain to a general fluid) must be of the form (3), with the new $\mathcal{E}$ and $\mathcal{B}$ obeying an equivalent Faraday law. These constraints may be difficult to fulfill.

It is worth stating here that the formalism developed contains, as its limiting cases, almost all non-relativistic and relativistic models of dissipation-less isotropic fluids studied so far, including the relativistic MHD.[12–14]

We will, however, first dwell on the 2F formalism of the ElectroVortic field.

II. GENERAL PLASMA EQUATIONS

The main steps in the Electro-Vortic formalism of the covariant relativistic dynamics of a perfect isotropic charged fluid, developed in Refs. 9 and 10, may be summarized as follows. The essence of the dynamics is contained in the conservation law for each fluid

$$\partial_\nu\left(T^{\mu\nu}_{fluid}+T^{\mu\nu}_{EM}\right)=0, \tag{4}$$

where $T^{\mu\nu}_{fluid}=hU^\mu U^\nu+p\eta^{\mu\nu}$ is the energy-momentum tensor of the fluid, with enthalpy h and pressure p, a four-velocity U^μ in a Minkowsky spacetime $\eta^{\mu\nu}$, and $T^{\mu\nu}_{EM}$ is the energy-momentum tensor of the electromagnetic field.[16] Equation (4) for each fluid is equivalent to

$$\partial_\nu(hU^\mu U^\nu)+\partial^\mu p=qnF^{\mu\nu}U_\nu, \tag{5}$$

where q is the electric charge of the particle, n is the density, and $F^{\mu\nu}$ is the electromagnetic field tensor. Also, each plasma specie obeys its own continuity equation

$$\partial_\mu(nU^\mu)=0\,. \tag{6}$$

The Maxwell equations,

$$\partial_\nu F^{\mu\nu}=4\pi\sum_i q_i n_i U_i^\mu, \tag{7}$$

provide the closure (where the sum is over all the species). Equations (5), (6) (for all species), and (7) complete the plasma dynamics.

A. Antisymmetric unified form of plasma equations

It is rather remarkable[9,10,15] that plasma equations can be written in terms of fully antisymmetric tensors; this requires casting the fluid forces (Vortical–Thermal) in electromagnetic clothing. We can define a fully antisymmetric fluid tensor field as

$$S^{\mu\nu}=\partial^\mu(fU^\nu+\mathcal{Z}\partial^\nu\sigma)-\partial^\nu(fU^\mu+\mathcal{Z}\partial^\mu\sigma), \tag{8}$$

in complete analogy with the electromagnetic field tensor $F^{\mu\nu}$. The antisymmetric tensor (8) contains the relativistic thermal and fluid properties of the plasma. Here, $f=h/mn$ is the enthalpy density per mass m, which depends on the plasma temperature T. The scalar entropy density σ, related to the other thermodynamical quantities by

$$T\partial^\mu\sigma=\frac{1}{n}\partial^\mu p-m\partial^\mu f, \tag{9}$$

is conserved along the flow lines

$$U^\mu\partial_\mu\sigma=0\,. \tag{10}$$

Lastly, the scalar quantity $\mathcal{Z}$ is related to the plasma temperature, and it satisfies

$$U^\mu\partial_\mu\mathcal{Z}=\frac{T}{m}\,. \tag{11}$$

Making use of all these exact properties, the entire dynamics becomes unified and is expressible in terms of the weighted Electro–Vortical field (EV) tensor

$$\mathcal{M}^{\mu\nu}=F^{\mu\nu}+\frac{m}{q}S^{\mu\nu}=\partial^\mu\Pi^\nu-\partial^\nu\Pi^\mu, \tag{12}$$

which signifies the weight of the fluid (through the mass m) and electromagnetic forces (through charge q). It is constructed as the four curl of the generalized potential

$$\Pi^\mu=A^\mu+\frac{mf}{q}U^\mu+\frac{m\mathcal{Z}}{q}\partial^\mu\sigma\,. \tag{13}$$

The plasma dynamical equation (5) may now be written elegantly and succinctly as

$$qnU_\nu\mathcal{M}^{\mu\nu}=0\,. \tag{14}$$

This equation is a profound expression of the fact that the fluid and the electromagnetic forces can be treated at par and are formally unified in the EV. Any general theorem on this dynamics, therefore, must involve only the EV field and not just the electromagnetic part. The EV field $\mathcal{M}^{\mu\nu}$ is a fully antisymmetric tensor of the second-rank (a generalization of the electromagnetic tensor $F^{\mu\nu}$).

In fact, the structure of (14) readily reveals the fundamental features of the fluid dynamics through the logical chain:

(1) The three vector (and the only independent) component of (14) is nothing but an ideal Ohm's law for the given fluid (repeated for convenience with the species index k suppressed)

$$\mathcal{E}+\mathbf{u}\times\mathcal{B}=0, \tag{15}$$

with the obvious identification $[\mathcal{E}^i=\mathcal{M}^{0i},\mathcal{B}^i=\mathcal{M}^{jk}$ with i, j, k cyclic] for $\mathcal{E}$ and $\mathcal{M}$ the electric and magnetic parts of the EV field.

(2) Since an ideal Ohm's law implies the connectivity of $\mathcal{B}$ with the fluid velocity field $\mathbf{u}$, a Connection theorem for the individual fluid is an ineluctable consequence of the dynamics controlled by (14). This implicit result was explicitly demonstrated for several variations in Refs. 4, 5, 7, and 8.

(3) The EV unification, thus, not only establishes the connectivity, it also gives full expression for the generalized field $\mathcal{B}$ that is connected to the flow field of the given fluid. One could, of course, trivially express the connected $\mathcal{B}$ in terms of the 1F variables, the magnetic field, the current, and the mass flow, remembering, however, that $\mathcal{B}$ is connected to the given fluid and not to the mass motion.

(4) Within the idealized dynamics, the connectivity (existence of a CT) is trivially guaranteed. When some non-ideal effects are introduced to violate it, it is the lines of $\mathcal{B}$ for each fluid that reconnect; the reconnection of the lines of the magnetic field is not even a meaningful proposition since these lines were never connected to the flow streamlines. It is only in the simplest MHD models that $\mathcal{B} \approx \mathbf{B}$, and magnetic reconnection is a meaningful concept. In fact, if the concept of reconnection is to be meaningfully studied in dynamics more general than MHD, we must embrace these new composite fields.

B. Relativistic two-fluid plasma in one-fluid (1F) variables

We will now manipulate the EV formalism pertaining to a two species plasma of opposite charged elements with masses m_+ and m_-. The positive and negative charged fluids, labelled by the symbols + and –, respectively, obey

$$en_+ U_{\nu+}\mathcal{M}_+^{\mu\nu} = 0\,, \qquad -en_- U_{\nu-}\mathcal{M}_-^{\mu\nu} = 0, \tag{16}$$

where e is the elementary charge. Let us now go over to the 1F variables, the (total) mass flow plasma four-velocity $\mathbf{U}^\mu$, and the four-current J^μ

$$\mathbf{U}^\mu = \frac{1}{\rho}\left(m_+ n_+ U_+^\mu + m_- n_- U_-^\mu\right), \qquad J^\mu = en_+ U_+^\mu - en_- U_-^\mu, \tag{17}$$

where $\rho = m_+ n_+ + m_- n_-$, with the inverse relationship

$$U_\pm^\mu = \frac{n\mathbf{U}^\mu}{n_\pm} \pm \frac{m_\mp J^\mu}{emn_\pm}, \tag{18}$$

where now $n = \rho/m$ is the effective plasma density ($m = m_+ + m_-$). We notice that, by using individual continuity Eqs. (6), we find a composite conservation law for the mass flow velocity

$$\partial_\mu (n\mathbf{U}^\mu) = 0\,. \tag{19}$$

The 1F formulation is facilitated by introducing two composite tensors: the first

$$Z^{\mu\nu} = \frac{1}{2}\left(\mathcal{M}_+^{\mu\nu} - \mathcal{M}_-^{\mu\nu}\right) = \frac{1}{2e}\left(m_+ S_+^{\mu\nu} + m_- S_-^{\mu\nu}\right) \tag{20}$$

appears to be a purely fluid tensor ($F^{\mu\nu}$ is missing), while the second

$$D^{\mu\nu} = \frac{1}{2}\left(\mathcal{M}_+^{\mu\nu} + \mathcal{M}_-^{\mu\nu}\right) = F^{\mu\nu} + \frac{1}{2e}\left(m_+ S_+^{\mu\nu} - m_- S_-^{\mu\nu}\right) \tag{21}$$

is a mixed tensor related to both kinds of forces. By construction, these antisymmetric tensors can be put as

$$Z^{\mu\nu} = \partial^\mu \mathcal{Y}^\nu - \partial^\nu \mathcal{Y}^\mu, \quad \mathcal{Y}^\mu = \frac{1}{2}\left(\Pi_+^\mu - \Pi_-^\mu\right), \qquad D^{\mu\nu} = \partial^\mu \mathcal{P}^\nu - \partial^\nu \mathcal{P}^\mu, \quad \mathcal{P}^\mu = \frac{1}{2}\left(\Pi_+^\mu + \Pi_-^\mu\right), \tag{22}$$

in terms of four-potentials. In particular,

$$\mathcal{P}^\mu = A^\mu + \frac{\kappa_1}{2e} n\mathbf{U}^\mu + \frac{\kappa_2}{2e^2} J^\mu + \frac{1}{2e}\left(m_+ \mathcal{Z}_+ \partial^\mu \sigma_+ - m_- \mathcal{Z}_- \partial^\mu \sigma_-\right), \tag{23}$$

with

$$\kappa_1 = \frac{m_+ f_+}{n_+} - \frac{m_- f_-}{n_-}\,, \qquad \kappa_2 = \frac{m_+ m_-}{m}\left(\frac{f_+}{n_+} + \frac{f_-}{n_-}\right). \tag{24}$$

With the preceding definitions, the addition and subtraction of the two Eqs. (16), lead to

$$J_\nu D^{\mu\nu} + 2ne\,\mathcal{U}_\nu Z^{\mu\nu} = 0, \tag{25}$$

$$2ne\,\mathcal{U}_\nu D^{\mu\nu} + J_\nu Z^{\mu\nu} = 0, \tag{26}$$

where [$\Delta\mu = (m_+ - m_-)/m$]

$$\mathcal{U}^\mu = \mathbf{U}^\mu - \frac{\Delta\mu}{2ne} J^\mu \tag{27}$$

may be viewed as some effective velocity which is not the velocity of the mass flow. Equations (25) and (26) expressed in terms of 1F plasma variables (as distinct from the individual fluid variables) form the dynamical system that we analyze. It is easy to notice that Eq. (25) is the generalization of the corresponding MHD equation of motion, whereas Eq. (26) is to be identified with a general form of Ohm's law. For the convenience of the reader, we show in Appendix A that Eqs. (25) and (26) reduce to the well-known non-relativistic MHD limits.

We must realize that for the time being, we have lost the succinctness and formal elegance of the original description. We will now explore whether this 1F formulation teaches us something new and whether it helps us to relate, more directly, with earlier studies.

III. EXAMINING THE POSSIBILITY OF A GENERAL CONNECTION THEOREM

The generalized dynamics contained in (26)

$$\mathcal{U}_\nu D^{\mu\nu} = \Gamma^\mu, \tag{28}$$

where

$$\Gamma^\mu \equiv \frac{1}{2en} J_\nu Z^{\nu\mu} \tag{29}$$

does not have the form of an ideal Ohm's law. The presence of a nonzero Γ^μ on the r.h.s. of (28) makes it resemble a "resistive" Ohm's law. The thermal-vortical features of $Z^{\nu\mu}/2en$ provide an effective tensorial resistivity that does not vanish in general. Therefore, for this fluid with a velocity $\mathcal{U}_\nu$, there is no connection theorem for the magnetic component of $D^{\mu\nu}$. A more formal demonstration, following the procedure developed in Refs. 7 and 8, is displayed in Appendix B.

Under very specific conditions, some non-ideal effects can be put in the form of a general gauge fields allowing a unified form of the CT.[12] However, the general lack of a CT for the composite field $D^{\mu\nu}$ (notice that the special case of $\Gamma^\mu = 0$ reduces the system to its MHD limit) implies that it cannot remain frozen-in into the plasma for all times; the field lines associated with the "magnetic" part of $D^{\mu\nu}$ can change their topology; the field can be created, annihilated, or reconnected through the effective resistivity introduced by Γ^μ. The field $D^{\mu\nu}$, therefore, is not a restraining anchor and is a poor substitute for individual unified tensors $M^{\mu\nu}$ to describe the plasma dynamics.

One can conclude then that any attempt to construct new and exact conventional physically meaningful Connection theorems (that freeze appropriate generalized fields in the mass flows) will not succeed. Physically meaningful Connections of this class are already described in Section II A; only $\mathcal{B}_{\pm i} = \varepsilon_{ijk}\mathcal{M}_{\pm}{}^{jk}$ are frozen in the two fluids moving, respectively, with velocities $\mathbf{u}_\pm$.

Despite this setback, there may be some value in pursuing the project of finding some restricted class of plasma dynamics where a Generalized Connection Theorem (GCT) may exist, i.e., we may find a composite field frozen in a fluid transported by a velocity that does not correspond to either the single fluid or the mass flow velocities. Such a formal exercise is executed in Appendix C where we have worked out conditions under which the field $D^{\mu\nu}$ may be connected to a "fluid" transported by a rather complicated "velocity field."

IV. THE HELICITY PERSPECTIVE

Since an ideal Ohm's law provides a necessary and sufficient condition both for a conserved helicity and for the existence of a Connected field, we could earn a different perspective on the physical content of Sec. III via a discussion on the conserved helicities of the system. As mentioned previously, each fluid (in the 2F plasma formalism) has an independent conserved helicity. Our experience in Sec. III suggests that in the 1F formalism, we may not find a conserved helicity characteristic of the composite fluid.

For each fluid specie, one may construct a helicity four-vector[9,10,17]

$$\mathcal{K}^\mu_\pm = \Pi_{\nu\pm}\mathbb{M}^{\mu\nu}_\pm, \tag{30}$$

where $\mathbb{M}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}\mathcal{M}_{\alpha\beta}$ is the dual of the EV field (12), and Π_μ is the four-potential (13) for each specie. It can be readily shown that the helicity four-vector for each fluid $\mathcal{K}^\mu_\pm$ is conserved, i.e., it is divergence free

$$\partial_\mu \mathcal{K}^\mu_\pm = 0, \tag{31}$$

if and only if (14) is satisfied. A profound consequence is the conservation of the generalized helicity for each fluid

$$\mathcal{H}_\pm = \langle \mathcal{K}^0_\pm \rangle = \langle \mathcal{A}_\pm \cdot \nabla \times \mathcal{A}_\pm \rangle, \tag{32}$$

where $\langle\,\rangle = \int d^3x$, and $\mathcal{A}_\pm = \mathbf{A} \pm m_\pm f_\pm \gamma_\pm \mathbf{v}_\pm/e \pm m_\pm \mathcal{Z}_\pm \nabla\sigma_\pm/e$. In this way, the conserved helicities $\mathcal{H}_\pm$ pertain to the individual fluids. The generalized fields $\nabla \times \mathcal{A}_\pm = \mathbf{B} \pm (m_\pm/e)\nabla \times (f_\pm\gamma_\pm\mathbf{v}_\pm) \pm (m_\pm/e)\nabla\mathcal{Z}_\pm \times \nabla\sigma_\pm$, therefore, stay connected or frozen in its respective fluid.

When one goes to a 1F description, these conserved helicities must pertain because the essential features of a dynamics do not change just because we chose to write the equations differently. It is certainly true that out of the two conserved helicities, we could construct any two linearly independent combinations or alternatively, and because of Eq. (31), we could construct any linear combination of the helicity four vectors ($\mathcal{K}^\mu = \alpha\mathcal{K}^\mu_+ + \beta\mathcal{K}^\mu_-$) that will still lead to a conserved helicity. We will find, however, that such a combination will not have the structure of Eq. (30). Let us, for instance, examine the conserved helicity four-vectors, $\mathcal{K}^\mu_1 = (\mathcal{K}^\mu_+ + \mathcal{K}^\mu_-)/2$ and $\mathcal{K}^\mu_2 = (\mathcal{K}^\mu_+ - \mathcal{K}^\mu_-)/2$. After some straightforward algebra, these four vectors are expressible in 1F variables

$$\begin{aligned} \mathcal{K}^\mu_1 &= \frac{1}{2}\left(\mathcal{P}_\nu\mathbb{D}^{\mu\nu} + \mathcal{Y}_\nu\mathbb{Z}^{\mu\nu}\right), \\ \mathcal{K}^\mu_2 &= \frac{1}{2}\left(\mathcal{Y}_\nu\mathbb{D}^{\mu\nu} + \mathcal{P}_\nu\mathbb{Z}^{\mu\nu}\right), \end{aligned} \tag{33}$$

where $\mathbb{D}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}D_{\alpha\beta}$ and $\mathbb{Z}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}Z_{\alpha\beta}$ are the dual of the fields (21) and (20), respectively. It is equally straightforward to show that both helicity four-vectors (33) are conserved only if Eqs. (25) and (26) are valid. Anew, the two resultant conserved generalized helicities

$$\mathcal{H}_{1,2} = \langle \mathcal{K}^0_{1,2} \rangle, \tag{34}$$

expressed in 1F variables, are simply the linear combinations of the two single fluid helicities. However, Eq. (33) is not of the form Eq. (30); it is only the latter that links a conserved helicity with a connected field. Since there is no conservation theorem involving $\mathbb{D}^{0i} = \epsilon^{0ijk}D_{jk}$, the field D_{ij} does not remain connected.

V. IMPLICATIONS FOR RECONNECTION

Having shown that the only fields that remain connected during fluid transport (frozen-in condition) are the general fields $\mathcal{M}^{\mu\nu}_\pm$ corresponding, respectively, to the positively and negatively charged components of the plasma, one must wonder about the implications of this result for the highly investigated reconnection process.

This enquiry must be made in the backdrop of our demonstration that we could not find any new field that remains connected during the transport via a physically meaningful velocity like the bulk plasma motion. The special case of $\Gamma^\mu = 0$ leads to connected fields that are simple limits of $\mathcal{M}^{\mu\nu}_\pm$. One could invoke a different transport apart from the natural one (velocity of the individual fluid and the bulk velocity) and find a connected field (as it is performed in Appendix C) but such a procedure, though mathematically possible, does not lead to any additional understanding that is not contained in the rigorously and naturally connected fields $\mathcal{M}_{\pm}^{\mu\nu}$ (in fact it is their magnetic components that are frozen-in)

Since the ElectroVortic field $\mathcal{M}_{\pm}^{\mu\nu}$, a unified combination of the electromagnetic and thermal-vortical forces is considerably more complicated than $F^{\mu\nu}$, and the connected field is, commensurately, more complicated than the magnetic field. Thus, barring the simplest models (MHD and hall MHD), the magnetic field is not the central variable in the physics of reconnection because it is not the field that was connected in the first place. What replaces the magnetic field is the magnetic component (EV_m) of the generalized ElectroVortic field ($\mathcal{B}^i = \mathcal{M}^{jk}$ with i, j, k cyclic) that does, indeed, remain connected in the ideal dynamics of a hot relativistic plasma. Naturally, the EV_m associated with either plasma specie differs more and more from the magnetic field B^i as the plasma description departs further from MHD. At some stage, the conventional subject of magnetic reconnection may not remain a legitimate pursuit.

Since the magnetic field is not connected in a general plasma, the field lines can reconnect (destroying the field in some region) without any aid from resistive or other external processes; magnetic field annihilation is allowed in the ideal dynamics. In the exactly opposite scenario, magnetic field can be created (from zero value) without an external seed, entirely within an ideal dynamics.[10] The evolution of the magnetic component of the generalized ElectroVortic field and of the magnetic field will be determined by a comparison between their available energy. In Ref. 14, in the non-relativistic regime, it was shown that "magnetic reconnection" was possible if large gradients of the electron plasma fluid velocity or density are achieved, in agreement with numerical results of Ref. 11. This is a very important problem for Hamiltonian reconnection.[14] In our work, the relevant magnetic component of the generalized ElectroVortic field for each fluid is $\mathcal{B} = \mathbf{B} + (mf/q)\nabla\times(\gamma\mathbf{u}) + (\gamma/qn)\nabla p\times\mathbf{u}$. Since large gradients of relativistic fluid velocity or relativistic pressure (and their associated energy) will make the nonmagnetic part to be large (even larger than the magnetic part in some cases), large changes in magnetic energy (including the conventional magnetic reconnection) are possible consistent with the conservation of the helicities determined by the magnetic component of the generalized ElectroVortic field. Even when large gradients are available, the ability of a given plasma state to shed its magnetic energy to flow or thermal energy is dictated by the value of the plasma helicity invariants (32) and the electromagnetic helicity invariant

$$\begin{aligned}\mathcal{H}_+ &= \langle \mathcal{A}_+ \cdot \nabla\times\mathcal{A}_+\rangle,\\ \mathcal{H}_- &= \langle \mathcal{A}_- \cdot \nabla\times\mathcal{A}_-\rangle,\\ \mathcal{H}_{\rm EM} &= \langle \mathbf{A}\cdot\mathbf{B}\rangle.\end{aligned} \tag{35}$$

It was shown in Ref. 18 that the destruction of magnetic field is catastrophically possible (though the magnetic field is not connected) while maintaining the invariants of the system. This can occur in an unstable state where flow energy dominates over magnetic energy, and thus, the system goes through a catastrophe, converting much of its magnetic energy into flow energy.[18] The dynamics of such transformations in a fully relativistic plasma will be explored in the future work.

The ElectroVortic formulation, emphasizing the "union" of electromagnetic and thermal-fluid forces, forces us to have a new look at the classical fields of magnetic reconnection and magnetic generation. Strictly speaking, ideal dynamics is perfectly capable of dealing with changes in the magnetic field (including field topology) that can be caused by its interaction with the thermal-vortical fields without invoking any additional forces. Perhaps, when dealing with the physics regimes beyond MHD, one must first emphasize the internal processes that convert electromagnetic into vortical-thermal energy and vice versa.

APPENDIX A: NON-RELATIVISTIC MHD LIMIT

Equations (25) and (26) can be shown to have the known non-relativistic MHD limits. For $m_- \ll m_+$ ($\Delta\mu \approx 1$), and constant densities $n_+ \sim n_- \sim n$, the plasma velocity

$$\begin{aligned}\mathbf{U}^\mu &\approx U_+^\mu + \frac{m_-}{m_+ en}J^\mu \approx U_+^\mu\,,\\ U_-^\mu &\approx \mathbf{U}^\mu - \frac{1}{en}J^\mu\end{aligned} \tag{A1}$$

is essentially that of the massive fluid. Also, in this limit, all the thermal-inertial effects are neglected ($J_\nu Z^{\mu\nu} \approx 0$ and $J_\nu D^{\mu\nu} \approx J_\nu F^{\mu\nu}$), and thus, Eqs. (25) and (26) become simply

$$J_\nu F^{\mu\nu} + 2ne\,\mathbf{U}_\nu Z^{\mu\nu} = 0, \tag{A2}$$

$$\mathbf{U}_\nu D^{\mu\nu} - \frac{1}{2ne}J_\mu F^{\mu\nu} = 0\,. \tag{A3}$$

Thus, the momentum equation (A2) becomes

$$nm_+\mathbf{U}^\nu\partial_\nu\mathbf{U}^\mu \approx J_\nu F^{\mu\nu} - \partial^\mu p, \tag{A4}$$

with $p = p_+ + p_-$. This equation reduces to its well-known non-relativistic MHD limit

$$nm_+(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = \mathbf{J}\times\mathbf{B} - \nabla p, \tag{A5}$$

as $U^\mu \to (1,\mathbf{v})$, and $J^\mu \to (0,\mathbf{J})$. Similarly, Ohm's law (A3) is now

$$\left(\mathbf{U}_\nu - \frac{1}{2en}J_\nu\right)F^{\mu\nu} - \frac{m_+}{2e}\mathbf{U}^\nu\partial_\nu\mathbf{U}^\mu \approx 0\,. \tag{A6}$$

By using (A4), we get

$$\left(\mathbf{U}_\nu - \frac{1}{en}J_\nu\right)F^{\mu\nu} + \frac{1}{en}\partial^\mu p_- \approx 0, \tag{A7}$$

which simplifies to the standard non-relativistic limit

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} - \frac{1}{en}\mathbf{J}\times\mathbf{B} + \frac{1}{en}\nabla p_- \approx 0\,. \tag{A8}$$

APPENDIX B: CONNECTION EQUATION FOR FIELD $D^{\mu\nu}$

We apply the operator $\varepsilon^{\alpha\beta\lambda\zeta}\varepsilon_{\alpha\beta\gamma\mu}\partial^\gamma$ to Eq. (28), where $\varepsilon^{\mu\nu\alpha\beta}$ is the Levi-Civita symbol; we obtain

$$\frac{d}{d\tau}D^{\lambda\zeta} = \partial^\lambda \mathcal{U}_\nu D^{\zeta\nu} - \partial^\zeta \mathcal{U}_\nu D^{\lambda\nu} - \partial^\lambda \Gamma^\zeta + \partial^\zeta \Gamma^\lambda, \tag{B1}$$

where $d/d\tau = \mathcal{U}^\mu \partial_\mu$ is the covariant convective derivative along the proper time τ defined by the transported velocity $\mathcal{U}^\mu$. As any plasma element is transported by a four-velocity $\mathcal{U}^\mu$, we can introduce the spacelike event-separation four-vector $dl_\mu = x'^\mu - x^\mu$ between two different elements of the plasma[4,5,7,8] that fulfill

$$\frac{d}{d\tau}dl^\mu = dl^\nu \partial_\nu \mathcal{U}^\mu . \tag{B2}$$

Now, we can calculate the variations of the four-vector $dl_\lambda D^{\lambda\zeta}$. Using Eqs. (B1) and (B2), we find

$$\frac{d}{d\tau}\left(dl_\lambda D^{\lambda\zeta}\right) = -\left(dl_\lambda D^{\lambda\nu}\right)\partial^\zeta \mathcal{U}_\nu + dl_\lambda\left(\partial^\zeta \Gamma^\lambda - \partial^\lambda \Gamma^\zeta\right). \tag{B3}$$

Analogous to an ideal MHD,[4,5] when the CT is valid, it tells that if $dl_\lambda D^{\lambda\zeta} = 0$ initially, then it will remain so for subsequent times. However, when more general physical effects are included into the one-fluid plasma dynamics, and the plasma is transported with velocity $\mathcal{U}^\mu$, Eq. (B3) predicts that it is not possible to fulfill the CT, as thermal and kinematic quantities introduce source terms, referring to the last two terms in (B3). This only can occur when $\Gamma^\mu = 0$ in ideal dynamics.

APPENDIX C: A POSSIBLE GENERALIZED CONNECTION THEOREM

The most simple generalization that can be performed requires to define the plasma transportation four-velocity as $\mathcal{U}^\mu + \Lambda^\mu$, as composed by the generalized fluid velocity (27), and a velocity field Λ^μ that allow us to prove the GCT.[7,8] The velocity field Λ^μ must contain information of the thermodynamical properties of the plasma.[7,8] Notice that if Λ^μ can be casted as

$$\Gamma^\mu = \Lambda_\nu D^{\nu\mu}, \tag{C1}$$

in such a way that Eq. (28) may be written as

$$(\mathcal{U}_\nu + \Lambda_\nu)D^{\mu\nu} = 0 . \tag{C2}$$

Thus, when the plasma is transported with velocity $\mathcal{U}_\nu + \Lambda_\nu$, the field $D^{\mu\nu}$ remains connected. This can be computed similarly, to the previous case. A GCT can be obtained by calculating the variations of $dl_\lambda D^{\lambda\zeta}$. We obtain

$$\frac{d}{d\tau}\left(dl_\lambda D^{\lambda\zeta}\right) = -\left(dl_\lambda D^{\lambda\nu}\right)\partial^\zeta(\mathcal{U}_\nu + \Lambda_\nu). \tag{C3}$$

Thereby, Eq. (C3), epitomizing a GCT, is a generalization of the CT for the non-ideal MHD plasmas. The previous idea is analogous to the one discussed in Refs. 7 and 8. The substantial content of GCT is that if $dl_\nu D^{\nu\mu}$ vanishes initially, then it will always remain so into the streamlines of the plasma flow with a generalized four-velocity $\mathcal{U}^\mu + \Lambda^\mu$. The connected field is not the magnetic field, but the more general structured field (21) that contains information about the plasma itself. These models reduce to the previous known limits of Refs. 7 and 8.

The GCT (C3) is well-defined, and it is very sensitive to the behavior of $\Lambda_\mu = (\Lambda_0, \mathbf{\Lambda})$. Defining $D^{0i} = \mathcal{E}^i$ and $D^{ij} = \epsilon^{ijk}\mathcal{B}_k$, the time and spatial components of the Eq. (C1) are

$$\begin{aligned} -\mathbf{\Lambda}\cdot\mathcal{E} &= \Gamma^0, \\ \Lambda_0\mathcal{E} - \mathbf{\Lambda}\times\mathcal{B} &= \mathbf{\Gamma}, \end{aligned} \tag{C4}$$

that are consistent with the constraint $\Lambda_\mu \Gamma^\mu = 0$. Solving the above equations, we can find the time and spatial components of Λ_μ. From Eq. (28), we have the constraint $\mathcal{U}_\mu \Gamma^\mu = 0$ that allows us to obtain that $\Gamma^0 = \mathcal{U}\cdot\mathbf{\Gamma}/\mathcal{U}^0$. Therefore, the solutions are

$$\begin{aligned} \Lambda_0 &= \frac{\mathbf{\Gamma}\cdot\mathcal{B}}{\mathcal{E}\cdot\mathcal{B}}, \\ \mathbf{\Lambda} &= -\alpha\,\mathcal{E} - \beta\,\mathcal{B} + \delta\,\mathcal{E}\times\mathcal{B}, \end{aligned} \tag{C5}$$

where

$$\begin{aligned} \alpha &= \frac{\mathbf{\Gamma}\cdot(\mathcal{E}\times\mathcal{B})}{|\mathcal{E}\times\mathcal{B}|^2}, \\ \beta &= \frac{\mathcal{U}\cdot\mathbf{\Gamma} + \alpha\,\mathcal{U}^0|\mathcal{E}|^2}{\mathcal{U}^0\mathcal{E}\cdot\mathcal{B}}, \\ \delta &= \frac{\mathbf{\Gamma}\cdot\mathcal{E} - \Lambda_0|\mathcal{E}|^2}{|\mathcal{E}|^2|\mathcal{B}|^2 - |\mathcal{E}\cdot\mathcal{B}|^2}. \end{aligned} \tag{C6}$$

Nonetheless, there is a problem with this procedure. The four-velocity Λ^μ does not correspond to any of the recognizable flow fields—either of the individual fluids or that of the mass motion. We can see in (C5) that Λ_μ depends on components of the field $D^{\mu\nu}$. Therefore, it is not an independent velocity of the fluid.

[1]L. Comisso and F. A. Asenjo, Phys. Rev. Lett. **113**, 045001 (2014).
[2]F. A. Asenjo and L. Comisso, Phys. Rev. Lett. **118**, 055101 (2017).
[3]W. A. Newcomb, Ann. Phys. (N.Y.) **3**, 347 (1958).
[4]F. Pegoraro, Europhys. Lett. **99**, 35001 (2012).
[5]F. Pegoraro, J. Plasma Phys. **82**, 555820201 (2016).
[6]F. A. Asenjo and L. Comisso, Phys. Rev. D **96**, 123004 (2017).
[7]F. A. Asenjo and L. Comisso, Phys. Rev. Lett. **114**, 115003 (2015).
[8]F. A. Asenjo, L. Comisso, and S. M. Mahajan, Phys. Plasmas **22**, 122109 (2015).
[9]S. M. Mahajan, Phys. Rev. Lett. **90**, 035001 (2003).
[10]S. M. Mahajan, Phys. Plasmas **23**, 112104 (2016).
[11]Y. D. Yoon and P. M. Bellan, Phys. Plasmas **24**, 052114 (2017).
[12]F. Pegoraro, Phys. Plasmas **22**, 112106 (2015).
[13]See for example A. Lichnerowicz, *Relativistic Hydrodynamics and Magnetohydrodynamics* (W. A. Benjamin, Inc., 1967); A. M. Anile, *Relativistic Fluids and Magneto-Fluid* (Cambridge University Press, 2005); L. Rezzolla and O. Zanotti, *Relativistic Hydrodynamics* (Oxford University Press, 2013); B. Punsly, *Black Hole Gravitohydromagnetics* (Springer, 2008); S. S. Komissarov, Mon. Not. R. Astron. Soc. **303**, 343 (1999); J. M. Martí and E. Müller, Living Rev. Relat. **6**, 7 (2003).
[14]E. Cafaro, D. Grasso, F. Pegoraro, F. Porcelli, and A. Saluzzi, Phys. Rev. Lett. **80**, 4430 (1998).
[15]J. D. Bekenstein, Astrophys. J. **319**, 207 (1987).
[16]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973).
[17]J. D. Bekenstein and E. Oron, Phys. Rev. D **18**, 1809 (1978).
[18]S. Ohsaki, N. L. Shatashvili, Z. Yoshida, and S. M. Mahajan, Astrophys. J. **570**, 395 (2002).