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Citation: Rev. Sci. Instrum. **83**, 123502 (2012); doi: 10.1063/1.4768536 View online: http://dx.doi.org/10.1063/1.4768536 View Table of Contents: http://rsi.aip.org/resource/1/RSINAK/v83/i12 Published by the American Institute of Physics.

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Development of large volume double ring penning plasma discharge source for efficient light emissions

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(Received 22 June 2012; accepted 6 November 2012; published online 4 December 2012)

In this paper, the development of large volume double ring Penning plasma discharge source for efficient light emissions is reported. The developed Penning discharge source consists of two cylindrical end cathodes of stainless steel having radius 6 cm and a gap 5.5 cm between them, which are fitted in the top and bottom flanges of the vacuum chamber. Two stainless steel anode rings with thickness 0.4 cm and inner diameters 6.45 cm having separation 2 cm are kept at the discharge centre. Neodymium (Nd₂Fe₁₄B) permanent magnets are physically inserted behind the cathodes for producing nearly uniform magnetic field of ~0.1 T at the center. Experiments and simulations have been performed for single and double anode ring configurations using helium gas discharge, which infer that double ring configuration gives better light emissions in the large volume Penning plasma discharge arrangement. The optical emission spectroscopy measurements are used to complement the observations. The spectral line-ratio technique is utilized to determine the electron plasma density in double ring plasma configuration is ~2 × 10¹¹ cm⁻³, which is around one order of magnitude larger than that of single ring arrangement. © 2012 American Institute of Physics. [http://dx.doi.org/10.1063/1.4768536]

I. INTRODUCTION

Penning plasma discharge (PPD) devices have been studied extensively during last few decades. This device was originally proposed by Penning as a low-pressure manometer (i.e., cold cathode or Penning gauge).¹ This has later been adopted to function as an ion source.²⁻⁵ Penning ion sources have been used for a variety of applications, such as sputtering and evaporation of surfaces, electromagnetic separation of isotopes, and fusion applications.^{6,7} The PPD devices are also attractive due to their extensive use as charged particles trapping,^{8,9} light emitting sources,¹⁰ etc. In the applications of light emitting sources, it has recently been proposed that the PPD device can be used to calibrate a vacuum ultraviolet (VUV) spectrometer-detector system,¹¹ which will be an easy and low cost method in comparison to the commonly known branching ratio method used in Tokamak plasmas.^{12,13} In fact, the PPD device can generate spectral radiations in the visible and VUV region simultaneously and visible radiations analysis can help in VUV intensity calibration once the desired basic plasma parameters are obtained from the visible spectra.

Keeping this in mind, a method on the basis of experimentally observed intensities of 9 spectral lines of neutral helium in the visible region was developed to infer a large number of plasma parameters simultaneously from a small geometry PPD source (i.e., SD-01 from Jobin- Yvon, France, a standard cylindrical source of 1 cm in diameter and 1 cm in length).¹¹ The collisional-radiative (CR) model of atomic data and analysis structure (ADAS) code and database¹⁴ was used for this analysis. The electron density, electron temperature, ground-state atom, and ion densities and also the triplet metastable state (2 ³S) density were the parameters thus estimated.¹¹ The derived plasma parameters were then used to obtain the absolute intensities of a few lines in the vacuum ultraviolet (VUV) region. The same were compared with the observed VUV spectral lines, recorded simultaneously with the visible lines, using a VUV spectrometer for which intensity calibration was not available. This has facilitated to determine the calibration factors for three spectral lines in VUV region.¹¹ It is to be mentioned that though the spectral match for experimentally observed spectral lines and predicted synthetic spectra was very good, under the optical thin plasma condition, the estimated electron plasma temperature from the used PPD source was found rather high $(\sim 40 \text{ eV})$. At such a temperature, neutral helium may not survive that shows a discrepancy in temperature estimation.

In the above reference, calculations were carried out assuming optically thin and diffusion less plasma conditions that may not necessarily be correct. We followed the optically thin plasma approximation mostly used for model simplification, which is very near homogeneous along the line-of-sight and remains in steady-state for the duration of the observation. However, if the plasma is optically thick for the radiation coming from the plasma, the re-absorption (opacity) of the photons may cause wrong information about the estimated plasma parameters.^{15, 16} So, the discrepancy in temperature obtained in the results needed to be thoroughly investigated, which may be due to the effect of the opacity and also the diffusion of neutral and metastable states. This

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requires that the proposed method need to be reevaluated and/or the temperature estimation need to be cross-checked by other diagnostics for meaningful conclusion.

Langmuir probe is the often used metallic probe diagnostics for electron density and temperature in low pressure plasmas.^{17,18} However, in the case of small geometries of the Penning discharge device, where sheath-sheath intersection would be prominent, Langmuir probe diagnostic can lead erroneous results.^{18,19} Hence, we proposed to develop a large volume Penning discharge device with a facility to insert a Langmuir probe in the plasma and also to observe the effect of opacity and diffusion on the observed spectral lines by the means of spectroscopic analysis.

The main task in making the PPD volume larger for VUV spectrometer-detector system calibration is to get the brighter emission without obstacles. For this, in this paper, a simple large dimension double ring Penning discharge device has been developed for efficient light emissions. However, when we stayed with single anode ring, we got worse confinement and also spectral emission saturated at higher pressures. In recent past, some efforts have been made to avoid saturation at higher gas pressures in PPD sources.^{20,21} These approaches are more complex than the one taken under the present study. Furthermore, certain other complex geometries of PPD sources have also been proposed in which confinement is much more important than the light emission.^{22,23} In fact, confinement was so good that they contained nonneutral plasmas at substantial densities. However, the geometries used were having much larger and precision-shaped anodes, nearly closing the gap between the cathodes, which is not appropriate for the application we are in need to get the light out of the PPD source. Thus, the double anode ring is a compromise between optical confinement and anode transparency for effective spectroscopic measurements and Langmuir probe insertion inside the large volume PPD source.

In a Penning discharge, the plasma is produced between two end cathodes and a shell anode, with power derived from DC high voltage power supply. The electron confinement is maintained very high in this geometry by end cathodes (electrostatic confinement) and a pair of permanent magnets, which produces an axial magnetic field (magnetic confinement). The PPD produces a large number of bright emission lines even at low pressures and the spectral distribution depends on the combination of discharge gas, applied magnetic field, applied power, cathode material, etc. Although Penning discharge sources have been used for many applications and different sources have been researched, 1-10, 20-23 many of these previous designs are complex, expensive, or developed for specific application. Many references do not thoroughly describe the source dimensions, making it difficult to construct and implement in different applications.

Our work describes a simple low-cost Penning discharge source that is fabricated with fundamental machining skills and normal laboratory accessories. It is being developed as radiating source for VUV-spectrometer detector system calibration.¹¹ The aim here is to describe the development and operation of the device. A comparison is made between single ring arrangement and double ring arrangement of PPD source through experiments and simulations. It is found that for large dimension discharge, the double ring operation of the developed device gives better plasma discharge current and radiations.

II. EXPERIMENTAL SET-UP

The schematic view of the experimental setup is shown in Fig. 1. The vacuum chamber is made of SS 304 L material. The chamber size is $20 \times 20 \times 45$ cm³. The main body of vacuum chamber is made by four rectangular plates and two end square plates. Nine ports are available in the chamber for different purposes. There are two ports on the top rectangular plate, one is connected to the vent valve and other one is connected to the high voltage feed-through to provide electrical connection to the anode. Each side plates of the chamber have three ports. One port is used to connect the cold cathode gauge (Alcatel Vacuum Technology, France). Two viewing ports are available for optical measurements. One port on the end square plate is available for inserting Langmuir probe inside the chamber. Pumping system is connected to vacuum chamber through other end vacuum port. Rotary and Turbo Molecular Pumps (Alcatel Vacuum Technology, France) are used to evacuate the chamber up to base pressure ~ 1 $\times 10^{-6}$ mbar. The gas pressure inside the chamber is regulated during the experiment by the high precision needle valve. The valve is mounted on the top rectangular plate.

The Penning configuration is shown in Fig. 2. It consists of two cylindrical end cathodes of radius 6 cm with a gap 5.5 cm, which are fitted in the top and bottom flanges of the vacuum chamber. Neodymium $(Nd_2Fe_{14}B)$ permanent magnets are physically inserted behind the cathodes for producing nearly uniform magnetic field of 0.1 T at the center. Two holes (0.5 cm) diagonally are machined on the inner side of the top rectangular plate. The Teflon tubes are mounted in these holes. Stainless steel rods of 1 cm diameter are used to support the anode ring through the Teflon tubes to prevent the plasma from unwanted high voltage arcing. For the double ring configuration of Penning discharge (see Fig. 2(a)), two anode rings with single connections are placed between two cylindrical end cathodes. On the other hand, for single ring configuration, a single ring with single connection is used (see



FIG. 1. Schematic view of the experimental setup of the Penning plasma discharge source.



FIG. 2. Schematic view of the electrode geometry of the Penning plasma discharge source for (a) double and (b) single ring configuration.

Fig. 2(b)). The anode rings are 0.4 cm thick and inner diameters are 6.45 cm each. A running water connection (10 l/min) is also arranged inside the cathodes for the steady operation of the device. Water-cooling connections are made from the top and bottom of the cathodes.

The system is first evacuated up to $\sim 10^{-6}$ mbar base pressure and then filled by helium gas up to a working pressure $\sim 10^{-5}$ mbar to 10^{-3} mbar. A high voltage DC power supply (2.5 kV/1A) is used between the cathodes and anode rings, which provides energy for the gas ionization and sustenance. For optical emission spectroscopy measurement, spectra are recorded using Acton SP series Spectrograph (SP-2500i PI, Action, USA) having focal length 500 mm, spectral resolution 0.05 nm, and gratings 1200 g/mm blazed at wavelength 500 nm. Entrance and exit slit width was 20 μ m and exposure time was 1 s. The light emitted from plasma is conveyed to the entrance slit of the spectrograph through a set of optics including optical fiber cable to image the light. The plasma is viewed by the optical fiber from a side port through a glass window. After dispersion of the light from the spectrograph, the emergent light is imaged by the Digital CCD camera (256BR PI, Action, USA) and data are stored in the computer.

III. RESULTS AND DISCUSSION

A. Simulations of vacuum electric and magnetic fields of the PPD source

To study the vacuum electric field in the designed Penning plasma discharge device, we have used AMAZE simulation code (Field Precision Inc., USA). The program solves Poisson's equation and continuity equation and provides a solution for quantities such as electric field and electric potential distributions. For simulation, first of all we have produced a mesh of desired geometry. The program generates the conformal triangular meshes for any 3-D system geometry. The goal is to divide the system into a number of small pieces or node elements. In this program, it is assumed that the physical properties of materials (such as dielectric constant) are uniform over the volume of the element so that the boundaries of the elements must be as close as possible to the boundaries of the physical object. In the mesh, each section of the geometry is assigned with different regions such as vacuum, conducting electrodes. After forming the desired mesh, the 3-D electric field simulation has been carried out at the applied voltage ~500–2500 V and dielectric constant $\varepsilon_{\rm r} = 1$. Figs. 3(a) and 3(b) show a cross-sectional view of the 3-D electric field



FIG. 3. The 3-D cross-sectional view of the electric field distribution in PPD source at 2 kV applied voltage for (a) double and (b) single ring configurations.

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FIG. 4. The 3-D view of the developed PPD geometry for simulation.

simulation in PPD source at 2 kV applied voltage for double and single ring configurations.

The electric field in the single ring configuration is zero at the center of the ring while in double ring the field is zero in between the two rings. Though the field lines are drawn for clarity and comparison, this electric field distribution may not necessarily be similar during the plasma discharge. The vacuum magnetic field profile has also been drawn for the developed PPD geometry using BfieldMsetup simulation code,²⁴ which shows uniform axial magnetic field in the discharge region between two cathodes.

B. Simulation of electric and magnetic fields in the PPD during plasma discharge

To study the discharge phenomenon and dynamics of charge particles in the PPD source, a particle-in-cell simulation code "VORPAL" (Tech-X Corporation, USA) is used. Using this code, three-dimensional electrode geometry is developed as shown in Fig. 4.

The section of the geometry is assigned with different regions such as gridded region (vacuum chamber), conducting electrodes as upper and lower cylinders (end cathodes) where magnets are fitted inside the cylinders, and two anode rings fitted at the discharge center between these end cathodes. The developed geometry describes the structural design of the PPD used in 3-D simulation, which is similar to the experiment. The input parameters are: applied voltage (2 kV), gas type (helium), working pressure $(1 \times 10^{-3} \text{ mbar})$, and magnetic field (0.1 T). During the plasma discharge, the obtained electric field profile in the PPD source is shown in Figs. 5(a) and 5(b) after 1 μ s time for both double and single ring configurations. Here, center of the each anode ring is taken as the origin.

It is evident that the applied electric field is localized near the anode rings during the plasma discharge in the PPD source, which was diffused till cathodes in the case of vacuum. Therefore, for large size PPD source, the chances of escaping ions in the cathode region would be very high in the case of single ring configuration. So, the introduction of another ring would detain the charge particles better within the large dimension cathode geometry. The applied axial magnetic field holds the electrons within the ring region and has been observed unchanged during plasma discharge from the simulations. From simulations, the accumulation of charge particles in the discharge geometry for single and double ring configurations is shown in the Figs. 6(b) and 6(a) after 1 μ s of plasma discharge. It is revealed from this figure that the ions diffusion in case of single ring configuration is more towards the cathodes while there is a small diffusion across the rings in double ring configuration (see Figs. 6(b) and 6(a)). More diffusion towards the cathodes would lead to more loss of ions and also the sputtering of cathodes at higher applied voltages. The sputtered material perturbs the plasma and also not useful for the performance of the source.

It is most obvious to see that in Penning type plasma discharges the ions play an important role in sputtering and secondary emissions whereas the electrons gyrating inside



FIG. 5. Electric field profile in the PPD source after 1 μ s time in the plasma discharge at 2 kV applied voltage for (a) double and (b) single ring configurations.



FIG. 6. Accumulation of charge particles (green dots show electrons and red dots show ions) in the discharge geometry at 1 μ s time and 2 kV applied voltage for (a) double and (b) single ring geometry.

the ring create more excitation. The same is evident from Figs. 6(a) and 6(b). The electron confinement in the double ring configuration of discharge volume is better than that of the ions (see Fig. 6(a)) while in the single ring configuration, the ions confinement within the discharge volume is better (see Fig. 6(b)). However, in double ring configuration the electric field distribution in the discharge volume is more than that of single ring configuration during the plasma discharge (see Figs. 5(a) and 5(b)). Due to this fact, the ions diffuse more towards the cathodes in the single ring configuration and create more sputtering. Whereas in double ring configuration and further the path length enhances due to axial magnetic field, which can lead multiple excitation and de-excitation to have enhanced radiations.

C. Electrical discharge analysis of the PPD source

For the plasma discharge characterization of the developed PPD source, the current-voltage characteristics have been obtained for double and single ring configurations at different working pressures and results are shown in Figs. 7(a) and 7(b). Fig. 7(a) shows that in the double ring configuration, the discharge current increases almost linearly with the applied voltage. However, the discharge current for single ring configuration after voltage ~ 1.5 kV becomes almost constant at lower pressures as shown in Fig. 7(b).

Also the breakdown voltages at different pressures for double ring arrangement are lower than that of the single ring, which is shown in Fig. 8. It has been further seen that the plasma discharge current at higher applied voltages and fixed operating pressure in case of single ring configuration is lower than that of double ring configuration. Hence, for enhanced plasma discharge operation in large volume, the double ring configuration of the PPD source appears better.

D. Optical emission discharge analysis of the PPD source

For equilibrium parameter measurements and radiation discharge analysis of PPD source, optical emission spectroscopy (OES) diagnostic technique is used. Fig. 9. shows



FIG. 7. The variation of discharge current at different applied voltages and for different working pressures (a) double and (b) single ring configuration.

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TABLE I. Electron density estimations for different discharge pressures and fixed discharge current in single and double ring configurations.

Discharge current 15 mA Working gas pressure (mbar)	Double ring		Single ring	
	9.0×10^{-4}	1.0×10^{-3}	$9.0 imes 10^{-4}$	1.0×10^{-3}
Observed line ratio (He I 667.81 nm/He I 728.13 nm) Predicted density (cm ⁻³)	$1.399 \\ \sim 2 \times 10^{11}$	$1.261 \\ \sim 1 \times 10^{11}$	$1.046 \ \sim 2 \times 10^{10}$	$\begin{array}{c} 1.034 \\ \sim 2 \times 10^{10} \end{array}$

the intensity variation with respect to the discharge current of helium plasma in the PPD source for double and single ring configurations simultaneously. It is seen that the intensity of the spectral line HeI 728.13 Å (2¹P-3¹S) increases linearly with discharge current in double ring configuration at working pressure 9×10^{-4} . However, for single ring configuration, the intensity increases first and later decreases for further increase of the discharge current.

The enhancement of intensities of other wavelengths at working pressure 6×10^{-3} mbar and discharge current 5 mA is shown in Fig. 10. The similar behaviour has been observed at different working pressures with same experimental conditions. So, it can be inferred that double ring configuration is more suitable as compare to single ring configuration for larger light emissions in the large volume PPD source. Fig. 9 also suggests that the double ring configuration is capable of drawing higher plasma currents.

The line intensity ratio 667.81 nm $(2^{1}P-3^{1}D)/728.13$ nm $(2^{1}P-3^{1}S)$ of the observed spectra is used to determine the electron plasma density in the developed PPD source. The observed intensity line ratios for this pair are compared with the same line ratio predicted from the Collisional-Radiative (CR) model based calculations already reported earlier.^{25,26} The observed line ratios and predicted electron plasma densities are given in Table I. These results are obtained for the single and double anode ring configurations at fixed discharge current 15 mA and at two working gas pressures 9×10^{-4} mbar and 1×10^{-3} mbar, respectively. The obtain plasma electron densities in the developed PPD source for double and single



FIG. 8. Breakdown voltages at different working pressures for double and sing ring discharge operation.



FIG. 9. The intensity variation of the spectral line HeI 728.13 Å $(2^{1}P-3^{1}S)$ with discharge current for double and single ring configurations at different pressures.



FIG. 10. Intensities of the observed helium spectral lines for both configurations at fixed pressure 6×10^{-3} mbar and fixed discharge current 5 mA.

ring configurations are $\sim 2 \times 10^{11}$ cm⁻³ and $\sim 2 \times 10^{10}$ cm⁻³, respectively. This further shows that the double ring configuration in large volume PPD source produces better plasma discharge density than that of the single ring configuration.

IV. CONCLUSION

A large volume double ring Penning plasma discharge source has been developed. The electrical, emission, and simulation measurements suggest that the large volume double ring PPD device is capable of producing more intense plasma light emission and plasma density than that of single ring operation at same discharge conditions. During the plasma discharge, the electric field concentrates more near the anode rings and leakage of charge particles becomes more around the cathodes for single ring operation in the large dimension PPD source. The introduction of second ring in the large volume PPD source has increased confinement of the charge particles, which leads to more emissions. The obtained electron plasma density in double ring configuration in this source is one order of magnitude higher than that of single ring configuration and is $\sim 2 \times 10^{11}$ cm⁻³.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the Board of Research in Fusion Science and Technology (BRFST) for the financial support for this work (Project No. NFP/DAQ/A09/02). Thanks are also due to Dr. Chandra Shekhar, Director, CSIR-CEERI Pilani, Dr. P. Vasu, IPR, Gandhinagar, and Dr. V. Shrivastava, CSIR-CEERI Pilani for useful discussion and support to develop the device. The authors also acknowledge anonymous referee for valuable suggestions to improve the content of the paper.

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