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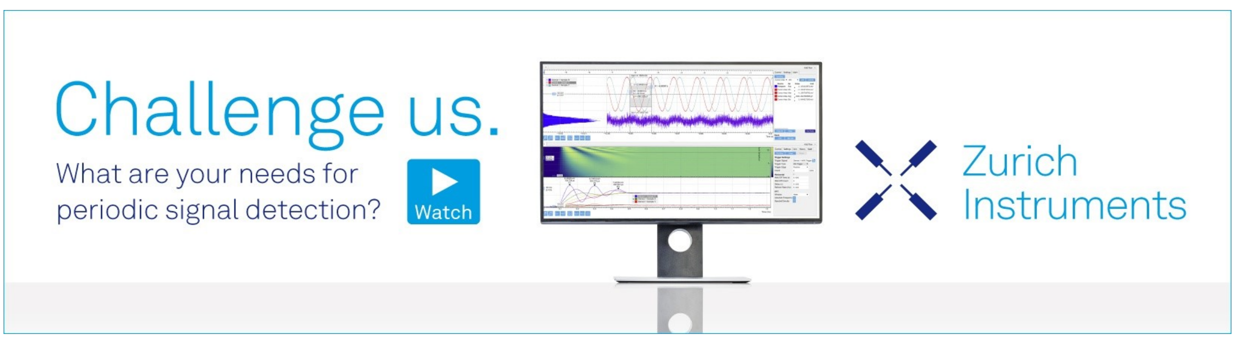

Convex Realizations Of Neural Codes In One Dimension

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Abstract. Neural codes are collective activity of the neurons which are electrically active cells in our brain. In this paper, We study the openness and closeness of the convex neural codes in one dimension. We discuss the possibilities of a convex code to be realized in dimension 1.That is conditions for the minimal embedding dimension of a neural code to be 1.

INTRODUCTION

The history of Neuropsychology which is the study of brain neurons got in to a new milestone when John O'Keefe discovered the place cells and also shared the Nobel price in Psychology or Medicine. A place cell is a neuron that fires when an animal is in a particular location relative to its environment.These particular locations called receptive fields are approximately convex.We can obtain binary neural code from the regions cut by these receptive fields.

After the discoveries of Hubel and Wiesel, O'Keefe and many others, Topology got a major role in Neuroscience. Carina Curto and Annie Shiu have discussed the constraints that a neural code to be convex in their paper.Rosen and Zhang in [1] had a combinatorial approach towards the convex codes in dimension 1.Franke and Muthiah have produced a nice result which provides the answer for the challenging question of the previous papers in this series.They establish that every binary code can be realized by convex sets.Also they raise some questions as conjectures in their article.

Mathematical Setup

In general we consider the collection of coactive neurons as coding units in neural populations.It has been observed experimentally that receptive fields arising from place cells are well approximated by sets that are not only convex but full dimensional. Thus it is usually assumed that receptive fields are open sets. A neural code is generally said to be convex if there exists open convex sets which realize the code.

. A binary code on n neurons is a collection of subsets $\mathscr{C}$ of the set n=$\{1,2,3,4......\}. The elements of C are called codewords.$
A code C is realizable in 1D if X is locally d-dimensional and there exists $\mathcal{U}$ with $U_i \subseteq X$ such that C = C ($\mathcal{U}$,X) in which case we say $\mathcal{U}$ realizes C

Main results

In a binary neural code of length n , If i always occurs with j, but j is independent of i then j th neuron is called supraneuron of i th neuron and i th neuron is called minineuron of j th neuron.

C=$\{1,3,12,123\}$. Three neurons are involved in this code. The code words related to the neuron 2 are 12 and 123. 2 always occurs together with 1. But the neuron 1 is independent of the neuron 2. Here neuron 1 is called the supraneuron of neuron 2 and neuron 2 is called the minineuron of neuron 1.
For any code there exist a open convex realization if and only if there is a closed convex realization.

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Let us assume that $U = \{I_1, I_2, I_3....I_n\}$ is a open convex realization of the code C where $C(U) = C$. Then we have to prove that there exist a closed convex realization $U^{'}$. First we have to fix ε Let us find the values $l_1, l_2, ..., l_n$ where l_i is the shortest distance between the intervals I_{i-1} and I_i.

So $\varepsilon = \big\{$ min l_i, if min $l_i \neq 0$,
1 if min $l_i = 0\big\}$

Then $a'_k = a_k + \varepsilon/3$ and $b'_k = b_k - \varepsilon/3$
We are shrinking I_k at open end points. Then $I'_k = [a'_k, b'_k]$ thus we can form $U' = \big\{I'_1, I'_2, I'_3....I'_n\big\}$ so that C(U')= C. Same way considering the other part, $U' = \big\{I'_1, I'_2, I'_3....I'_n\big\}$ where U' is a closed convex realization of C and also C(U')= C. Now to prove that there exist a open convex realization U .
Let us consider $a_k = a'_k - \varepsilon/3$ and $b_k = b'_k + \varepsilon/3$ we are extending at the closed end points. Then $I_k = [a_k, b_k]$ thus $U = \{I_1, I_2, I_3....I_n\}$ and C(U)=C.
Hence the proof.

WHY DONT WE INCLUDE ARBITRARY REALIZATION IN THE ABOVE THEOREM

In the paper"Every Binary code can be realized by Convex sets", Franke and Samuel proposed a conjecture that a code C has minimal embedding dimension 1 if and only if it has minimal convex embedding dimension 1. we eliminate the arbitrary case by the following example.

Example

Consider the Code C=$\big\{$12,13$\big\}$.
This code has only arbitrary realization in the minimal embedding dimension 1. Neither open nor closed realization is possible for this code in the minimal embedding dimension 1.

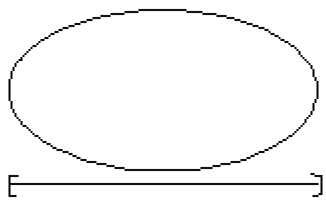
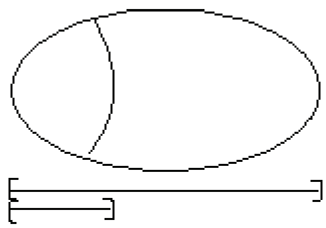
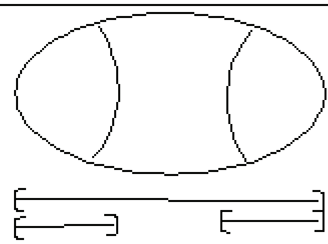
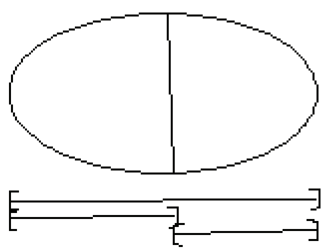
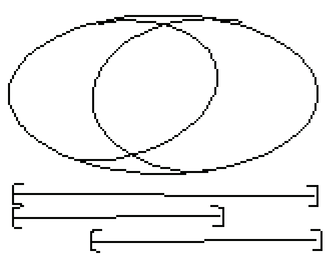
Discussion of different type of split ups of a set

To have a better understanding we just recollect all the possibilities of intersections of a set in 1 dimension.

The code has the minimal embedding dimension 1 if the number of elements in the code should be less than or equal to 2n where n is the number of neurons involved in the code.
A neuron in one dimension can introduce at the maximum 2 code words in the Code C. So, n neurons which are realized in one dimension will have maximum 2n code words.

Ordering of Neurons

As the firing of neurons is a consecutive action we can follow an order for the neurons . The neurons are arranged in an order. That is the response of first neuron is recorded as first code the second neuron's as the second and so on. We can state and prove the following theorem in the ordering perspective of neurons.
If the code C has the code word ij where i and j are not consecutive numbers but still the minimal embedding dimension is 1 then i th and jth neurons are not minineurons of any other neurons and the neurons lies between i th and j th neurons will be minineurons either of ith neuron or jth neuron not both.
Suppose that if the code C which is realizable in one dimensional has the code word ij .
We can prove this by the method of induction.
Case(i):

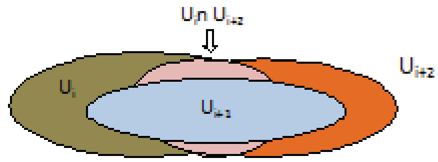
Sets with its intersections	Codes	Existing Realizations in minimal embedding dimension 1
	1	Open realization Closed realization Arbitrary realization
	12,2	Open realization Closed realization Arbitrary realization
	2,12,23,	Open realization Closed realization Arbitrary realization
	12,23	Arbitrary realization
	12,23,123	Open realization Closed realization Arbitrary realization

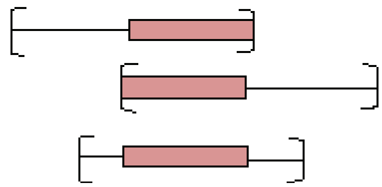
First let us prove the result for 1.

As i and j are nonconsecutive numbers there exists an integer such that, i<k<j.So, k=i+1 and j=i+2. Suppose that U_{i+1} is not a minineuron of U_i and also not a minineuron of U_{i+2} . A part of U_{i+1} lies in U_i and remaining lies in U_{i+2} as shown in the above figure.From the diagram its clear that the code word ij was not realized which is a contradiction to our assumption. Hence U_{i+1} will be minineuron of either U_i or U_{i+2}.

Case (ii):

Let us assume that the result is true for n and need to prove for n+1. Now we have n+1 neurons lying between i and





jth neuron. By our assumption n neurons are minineurons of either ith neuron or the jth neuron. We are left with the single n+1 th neuron, we can apply the result of case (i). Hence n+1th neuron is also a minineuron of either ith neuron or the jth neuron. Hence the proof.

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