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A novel technique to compute static and dynamic performance characteristics of aerostatic thrust bearing

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1. Introduction

Aerostatic thrust bearing are widely used in industrial metrology, grinding machines, semiconductor manufacturing equipment, high precision equipment and machines[1]. Bearing in these machineries prevents direct metal to metal contact and provide low friction environment, they provide almost frictionless environment to the machine. The stiffness and damping are a desirable property for various industrial application. Although stiffness and damping provided by these bearing is sufficient, there is a higher demand to improve stiffness and damping coefficient for precise operation. That is why, several experimental and theoretical studies pertaining the performance of an aerostatic thrust bearing have been carried out.[1-7].

These bearings have the capability to provide high accuracy, low heat generation and near-zero friction running characteristics[2]. Manufacturing error on aerostatic bearing surfaces generally causes deterioration in the performance of bearing performance[3, 4]. Manufacturing error can occur either on bearing surface or on a restrictor. Further, nano-fluctuation of objects may cause severe problem in precision based manufacturing system. Yoshimura[8] studied the cause of theses nano-fluctuation of aerostatic thrust bearing with surface restriction.

The aerostatic thrust bearing is usually designed to operate under parallel operation but thrust pad of bearing is tilted due several reasons such as manufacturing errors, thermal deformation and structural deformation. Tilt in thrust bearing results in a substantial deterioration in bearing performance[9]. Tilt reduced the bearing load carrying capacity, minimum air film thickness and increases air film temperature. A significant change in the bearing performance of the bearing is also observed.

Various types of recess shapes have been proposed for aerostatic thrust bearing. Yabe and his co-researchers[1, 5, 6] worked on annular and rectangular thrust bearings, but they focussed on running accuracy aspects only rather than effects on load and stiffness. The changes in load and stiffness occur in rectangular thrust bearings due to manufacturing error.

Tilt significantly alters the bearing performance. Therefore, in the present paper, the influence of the tilt parameter on the thrust bearing along with various recess shape has been studied. In the present work, elliptical, square, circular and rectangular type of recess have been considered[1]. Compressible Reynolds equation is nonlinear in pressure therefore, analytical methods cannot be used to find solution. Therefore, various numerical methods have been presented to find solution[10]. Ma et al.[11] used finite element method to solve compressible Reynolds equation for aerostatic thrust bearing. They employed 4^{th} order Runge-Kutta method and FEM are used to obtain time dependent behavior of aerostatic thrust bearing. They also compared stiffness of aerostatic bearing with different number of damping orifice. Zhou et al.[12] employed CFD approach to the numerical solution of aerostatic thrust bearings. In this method, they decomposed the solution domain into an inner region near the restrictor for Navier-stokes and Reynolds equations is used to solve air flow field. The results of approach had been compared with the numerical solution of aerostatic thrust bearing having cylindrical and rectangular recess. They proved that the hybrid approach can be used to efficiently model the complex flow behaviors near restrictor. They determined the pressure distribution near restrictor. Eleshaky [13] studied the effect of shock wave on the aerostatic thrust bearing by solving three dimensional average Reynolds equation. Sharma et al.[14] reported the effect of shape of recess on orifice compensated hydrostatic thrust bearing. The dynamic (stiffness and damping) coefficient of thrust bearing were studied by using grid method. Incompressible Reynolds equation for hydrostatic bearing also solved using FEM. The static and dynamic characteristics had been determined numerically. More recently Yadav et al.[15] developed a finite element formulation to compute static and dynamic performance parameter of gas lubricated journal bearing. They computed the gas film stiffness and damping coefficient and compared the finite element results with the results of perturbation method.

The open literature in the area of aerostatic thrust bearing reveals that there is no study that indicates the influence of tilt on aerostatic thrust bearing with different geometries of recess. Therefore, to bridge this gap in the literature, present study is planned to examine the performance of tilted pad aerostatic thrust bearings with different geometric shapes of recess. Furthermore, in past many attempts have been made to compute air stiffness and damping coefficient of aerostatic thrust bearing by using perturbation methods. The air film stiffness and damping characteristics of thrust bearing are studied by using dynamic grid method of ANSYS[16]. As the Reynolds equation governing the flow of compressible lubricant is nonlinear partial differential equation therefore, the computation of the stiffness and damping coefficient follow an iterative procedure. It requires lot of computational supports. Therefore, in the present work, a novel technique based on finite element formulation is suggested to compute air film stiffness and damping coefficient in aerostatic thrust bearing.

Nomenclature

A	Area, m^2
a	Radius of capillary, m
C	Air film damping coefficient, Ns/m
F	Resultant air film reaction $\left(\frac{\partial h}{\partial t} \neq 0\right)$, N
F_0	Resultant air film reaction$\left(\frac{\partial h}{\partial t} = 0\right)$, N
h	Local air film thickness, m
h_r	Reference air film thickness, m
h_0	Minimum clearance, m
p	Air film pressure, N/m^2
p_{oc}	Pocket pressure$\left(\frac{\partial h}{\partial t} = 0\right)$, N/m^2
p_s	Supply pressure, N/m^2
Q	Bearing flow, m^3/s
Q_R	Restrictor flow, m^3/s
n	Total number of nodes
n_t	Total number of elements
n_p	Number of pockets
n_l	Total number of nodes in an element
r_o	Outer radius of the circular thrust pad, m
S	Air film stiffness coefficient, N/m
t	Time, s
μ_r	Reference viscosity of the air, *Pa-s*

λ	Tilt parameter
p_a	Ambient pressure
p_s	Supplied pressure
ρ	Air viscosity
T	Air Temperature
g	Acceleration of gravity
κ	Isentropic expansion index of air
R_g	Gas constant

Non-dimensional parameters

$\bar{C}$	$\frac{Ch_f^3}{r_o^4\mu}$
$\bar{C}_{s2}$	$AP_s\phi\sqrt{\frac{2\rho}{P_s}} \times F_{orifice}$
$F_{orifice}$	$\frac{12\eta RgT}{c^3 p_a^2}$
$\bar{F}_0$	$\frac{F_0}{r_o^2 p_s}$
$\bar{F}$	$\frac{F}{r_o^2 p_s}$
$\bar{h}$	$\frac{h}{h_r}$
$\bar{A}$	$\frac{A_b}{A_c}$
$\bar{\dot{h}}$	$\frac{\partial\bar{h}}{\partial\bar{t}}$
$\bar{p}$	p/p_s
$\bar{Q}_R$	$\frac{12\mu}{p_s h_r^3}Q_R$

$\bar{Q}$	$\frac{\mu}{p_s h_r^3} Q$
$\bar{S}$	$\frac{S h_r}{r_o^2 p_s}$
$\bar{\mu}$	$\frac{\mu}{\mu_r}$
α	X/r_o
β	Y/r_o
$\bar{t}$	$t / \left(\frac{\mu r_o^2}{h_r^2 p_s} \right)$

Subscripts and superscripts

b	Bearing
e	e^{th} element
oc	Pocket
0	Steady state equilibrium
R	Restrictor
–	Corresponding dimensionless parameter
s	Supply pressure

Matrices

$[\bar{\boldsymbol{F}}\mathbf{1}], [\bar{\boldsymbol{F}}\mathbf{2}], [\bar{\boldsymbol{F}}\mathbf{3}], [\bar{\boldsymbol{F}}\mathbf{4}]$	Assembled air fluidity Matrix
$\{\bar{\boldsymbol{p}}\}$	Nodal pressure column Vector
$\{\overline{\mathbf{Q}}\}$	Nodal Flow Column Vector
$[\bar{\boldsymbol{R}}_{\boldsymbol{t}}]$	Right hand side column vector due to squeeze terms

2. Analysis

A schematic aerostatic thrust bearing using orifice compensator having circular recess shape has been shown in figure 1. The Reynolds equation governing the flow of compressible lubricant is expressed in non-dimensional as follows [7, 12, 17-19].

$$\frac{\partial}{\partial\alpha}\left(\frac{\bar{h}^3}{12}\bar{p}\frac{\partial\bar{p}}{\partial\alpha}\right)+\frac{\partial}{\partial\beta}\left(\frac{\bar{h}^3}{12}\bar{p}\frac{\partial\bar{p}}{\partial\beta}\right)=\frac{\partial(\bar{p}\bar{h})}{\partial\bar{t}} \tag{1}$$

Air film thickness is computed by using following expression[20]. Further, for a compressible lubricant, non-dimensional density may be considered as non-dimensional pressure.

$\bar{h}=\bar{h}_r+\bar{\lambda}\alpha$ where $\bar{\lambda}=\frac{\lambda r_o}{h_r}$ is defined as a tilt parameter. (2)

3. Finite element formulation

The Reynolds equation governing the flow of compressible lubricant is nonlinear in pressure and cannot be solved by analytical and direct numerical method. Therefore, in present work, nonlinear finite element method is used to compute the performance parameters of aerostatic thrust bearing. The finite element method is preferred due to its advantage over other numerical technique. Its solution is numerically more stable than other methods. The finite element discretization of air flow field is shown in figure 2. The air pressure variation over an element is computed as[17, 21, 22].

$$\bar{p}=\sum_{j=1}^{n_l^e}\bar{p}_jN_j \tag{3}$$

where, N_j is the nodal shape function and n_l^e is number of nodes in a quadrilateral element. Here value of n_l^e is four. Using Galerkin's orthogonality conditions and following usual assembly procedure of finite element method, linearized system of equation is written as following[23-25].

$$[\bar{F}1]\{\bar{p}\}=\{\bar{Q}\}+\dot{\bar{h}}[\bar{R}_t] \tag{4}$$

Here matrix $\bar{\mathbf{F}}\mathbf{1}$, $\bar{Q}$ and $\bar{R}_t$ are depend on the value of air film pressure and which is unknown. Therefore, a great care is required to initialize the value of pressure.

For stiffness computation the Eqn. (1) is differentiated with respect to $\bar{h}$ and following system equation is obtained

$$\frac{\partial}{\partial \bar{h}}\left(\frac{\partial}{\partial \alpha}\left(\frac{\bar{h}^3}{12}\bar{p}\frac{\partial \bar{p}}{\partial \alpha}\right)+\frac{\partial}{\partial \beta}\left(\frac{\bar{h}^3}{12}\bar{p}\frac{\partial \bar{p}}{\partial \beta}\right)=\frac{\bar{u}}{2}\frac{\partial(\bar{p}\bar{h})}{\partial \alpha}+\frac{\partial(\bar{p}\bar{h})}{\partial \tau}\right) \quad (5)$$

$$\frac{\partial}{\partial \alpha}\left(3\frac{\bar{h}^2}{12}\frac{\partial h}{\partial \bar{h}}\bar{p}\frac{\partial \bar{p}}{\partial \alpha}+\frac{\bar{h}^3}{12}\bar{p}\frac{\partial}{\partial \bar{h}}\frac{\partial \bar{p}}{\partial \alpha}+\frac{\bar{h}^3}{12}\frac{\partial \bar{p}}{\partial \alpha}\frac{\partial \bar{p}}{\partial \bar{h}}\right)+\frac{\partial}{\partial \beta}\left(3\frac{\bar{h}^2}{12}\frac{\partial h}{\partial \bar{h}}\bar{p}\frac{\partial \bar{p}}{\partial \beta}+\frac{\bar{h}^3}{12}\bar{p}\frac{\partial}{\partial \bar{h}}\frac{\partial \bar{p}}{\partial \beta}+\frac{\bar{h}^3}{12}\frac{\partial \bar{p}}{\partial \beta}\frac{\partial \bar{p}}{\partial \bar{h}}\right)=0 \quad (6)$$

$$[\bar{F}1+F3]\left\{\frac{\partial \bar{p}}{\partial \bar{h}}\right\}=\left\{\frac{\partial \bar{Q}}{\partial \bar{h}}\right\}-[F2]\{p\} \quad (7)$$

The above procedure is applied to compute air film damping coefficient of bearing. Therefore, Eqn.(1) is differentiated with respect to $\bar{\dot{h}}$.

$$\frac{\partial}{\partial \bar{\dot{h}}}\left(\frac{\partial}{\partial \alpha}\left(\frac{\bar{h}^3}{12}\bar{p}\frac{\partial \bar{p}}{\partial \alpha}\right)+\frac{\partial}{\partial \beta}\left(\frac{\bar{h}^3}{12}\bar{p}\frac{\partial \bar{p}}{\partial \beta}\right)=\frac{\partial(\bar{p}\bar{h})}{\partial \tau}\right) \quad (8)$$

$$\frac{\partial}{\partial \alpha}\left(3\frac{\bar{h}^2}{12}\frac{\partial h}{\partial \bar{\dot{h}}}\bar{p}\frac{\partial \bar{p}}{\partial \alpha}+\frac{\bar{h}^3}{12}\bar{p}\frac{\partial}{\partial \bar{\dot{h}}}\frac{\partial \bar{p}}{\partial \alpha}+\frac{\bar{h}^3}{12}\frac{\partial \bar{p}}{\partial \alpha}\frac{\partial \bar{p}}{\partial \bar{\dot{h}}}\right)+\frac{\partial}{\partial \beta}\left(3\frac{\bar{h}^2}{12}\frac{\partial h}{\partial \bar{\dot{h}}}\bar{p}\frac{\partial \bar{p}}{\partial \beta}+\frac{\bar{h}^3}{12}\bar{p}\frac{\partial}{\partial \bar{\dot{h}}}\frac{\partial \bar{p}}{\partial \beta}+\frac{\bar{h}^3}{12}\frac{\partial \bar{p}}{\partial \beta}\frac{\partial \bar{p}}{\partial \bar{\dot{h}}}\right)=\bar{p} \quad (9)$$

$$[\bar{F}1+F3]\left\{\frac{\partial \bar{p}}{\partial \bar{\dot{h}}}\right\}=R_t^e \quad (10)$$

The matrixes for e^{th} element are computed by using following expression[11]

$$\bar{F}1_{ij}^e=\int\int\frac{h^3}{12}\bar{p}\left[\frac{\partial N_i}{\partial \alpha}\frac{\partial N_j}{\partial \alpha}+\frac{\partial N_i}{\partial \beta}\frac{\partial N_j}{\partial \beta}\right]\partial\Omega \quad (11a)$$

$$\bar{F}2_{ij}^e=\int\int 3\frac{h^2}{12}\frac{\partial \bar{h}}{\partial \bar{Z}}\ \bar{p}\left[\frac{\partial N_i}{\partial \alpha}\frac{\partial N_j}{\partial \beta}+\frac{\partial N_i}{\partial \beta}\frac{\partial N_j}{\partial \beta}\right]\partial\Omega \quad (11b)$$

$$\bar{F}3_{ij}^e=\int\int\frac{h^3}{12}\left[\frac{\partial N_i}{\partial \alpha}\frac{\partial \bar{p}}{\partial \alpha}N_j+\frac{\partial N_i}{\partial \alpha}\frac{\partial \bar{p}}{\partial \beta}N_j\right]\partial\Omega \quad (11c)$$

$$\bar{F}4_{ij}^e=\int\int\frac{h^3}{12}(\bar{p}+N_j\bar{p}_j)\left[\frac{\partial N_i}{\partial \alpha}\frac{\partial N_j}{\partial \alpha}+\frac{\partial N_i}{\partial \beta}\frac{\partial N_j}{\partial \beta}\right]\partial\Omega \quad (11d)$$

$$Q_i^e=\oint\left\{\bar{p}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \alpha}\right)\vec{n}_1+\bar{p}\left(\bar{h}^3\frac{\partial \bar{p}}{\partial \beta}\right)\vec{n}_2\right\}N_i d\Gamma^e \quad (11e)$$

$$R_t^e=\int\bar{p}N_i\partial\Omega \quad (11f)$$

where $\vec{n}_1$ and $\vec{n}_2$ are directional vectors and i, j=1, 2 …n_l^e (number of nodes per element). Γ^e is the boundary of element and e^{th} is the element number. After computing the elemental matrices, they are assembled into global system of linearized equations[26].

3.1. Air flow rate from restrictor:

The air flow passing through the feeding orifice is dependent on the air density and supplied pressure and can be computed from the following equation [27]

$$\dot{m}_{in} = AP_s\phi\sqrt{\frac{2\rho}{P_s}}\,\Psi \tag{12}$$

$$\Psi = \begin{Bmatrix} \left[\frac{\kappa}{2}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}\right]^{1/2}, & \bar{p}_{oc} < \beta_k \\ \left\{\frac{\kappa}{\kappa-1}\left[(\bar{p}_{oc})^{\frac{2}{\kappa}} - (\bar{p}_{oc})^{\frac{\kappa+1}{\kappa}}\right]\right\}^{1/2} & \bar{p}_{oc} > \beta_k \end{Bmatrix} \tag{13}$$

where

$$\beta_c = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \tag{14}$$

A is the cross-sectional area of the feeding orifice, κ is isentropic expansion index, $\bar{p}_s$ is the supplying pressure, $\bar{p}_{oc}$ is pocket pressure of feeding orifice, ϕ is damping of orifice.

By combining all the terms of Eqn. (12), we get the eq(13) in modified form as.

$$\dot{m}_{in} = C_{s2}\Psi \qquad \text{where } C_{s2} = AP_s\phi\sqrt{\frac{2\rho}{P_s}} \tag{15}$$

In non-dimensional form above equation is expressed as.

$$\bar{\dot{m}}_{in} = \bar{C}_{s2}\,\Psi \tag{16}$$

where, $\bar{C}_{s2}$ is restrictor deign parameter and in the present work, value of $\bar{C}_{s2}$ has been taken from 0.5 to 1.5.

$$\bar{C}_{s2} = AP_s\phi\sqrt{\frac{2\rho}{P_s}} \times F_{orifice} \quad \text{where } F_{orifice} = \frac{12\eta RgT}{c^3 p_a^2}$$

4. Bearing performance parameter

In the present work, pocket pressure, air flow rate and load carrying capacity are presented as static performance parameter whereas air film stiffness and damping coefficients are presented as dynamic performance parameters.

4.1. Air film reaction ($\bar{F}_0$)

For the computation of air film reaction, integration of air film pressure is carried out on all elements and bearing pocket [24].

$$\bar{F}_0 = \bar{F}_0|_{contribution\ due\ to\ land\ area} + \bar{F}_0|_{contribution\ due\ to\ pocket\ area} \tag{17}$$

$$\bar{F}_o = \sum_{e=1}^{n_e} \left\{ \int_{-1}^{1}\int_{-1}^{1} \left(\sum_{i=1}^{n_l=4} \bar{p}_i N_i \right) |\bar{J}| d\xi d\eta \right\} + \sum_{i=1}^{i=n_p} \bar{A}_{oc} \bar{p}_{oc} \tag{18}$$

4.2. Air film Stiffness Coefficient

Applying differentiation on eq(1) with respect to variable $\bar{h}$ and applying Galerkin technique we get the following equation.

$$[\bar{F}1 + F3] \left\{ \frac{\partial \bar{p}}{\partial \bar{h}} \right\} = \left\{ \frac{\partial \bar{Q}}{\partial \bar{h}} \right\} - [F2]\{p\} \tag{19}$$

The above equation may be written as.

$$\left\{ \frac{\partial p}{\partial \bar{h}} \right\} = [\bar{F}1 + F3]^{-1} \left\{ \left\{ \frac{\partial \bar{Q}}{\partial \bar{h}} \right\} - [\bar{F}2]\{\bar{p}\} \right\} \tag{20}$$

As the matrices $\bar{\mathbf{F}}\mathbf{1}$, $\bar{\mathbf{F}}\mathbf{2}$ and $\bar{\mathbf{F}}\mathbf{3}$ are dependent on the pressure column vector $\{\bar{\mathbf{p}}\}$ cannot be computed easily. Hence, a iterative procedure has been performed in two step. In this simulation, the iteration has been performed on the on the air film pressure and derivative of air film pressure.

Air film stiffness coefficient is the summation of pressure derivatives on the area and it is computed as follows.

$$\bar{S} = \frac{\partial \bar{F}_0}{\partial \bar{h}} = \sum_{e=1}^{n_e} \left\{ \int_{-1}^{1}\int_{-1}^{1} \left(\sum_{i=1}^{n_l=4} \frac{\partial \bar{p}_i}{\partial h} N_i \right) |\bar{J}| d\xi d\eta \right\} + \sum_{i=1}^{i=n_p} \bar{A}_{oc} \frac{\partial \bar{p}_{oc}}{\partial \bar{h}} \tag{21}$$

4.3. Air film Damping Coefficient

The computation of air film damping has been performed by using following expression.

$$[\bar{\boldsymbol{F}}\mathbf{1}+\boldsymbol{F}\mathbf{3}]\left\{\frac{\partial\bar{\boldsymbol{p}}}{\partial\bar{\dot{h}}}\right\}=\left\{\frac{\partial\bar{\boldsymbol{Q}}}{\partial\bar{\dot{h}}}\right\}+\boldsymbol{R}_t^e \tag{22}$$

The above equation may be written as

$$\left\{\frac{\partial\bar{\boldsymbol{p}}}{\partial\bar{\dot{h}}}\right\}=[\bar{\boldsymbol{F}}\mathbf{1}+\boldsymbol{F}\mathbf{3}]^{-1}\left\{\left\{\frac{\partial\bar{Q}}{\partial\bar{\dot{h}}}\right\}+\left\{\frac{\partial R_t}{\partial\bar{\dot{h}}}\right\}-[\bar{F}2]\{\bar{p}\}\right\} \tag{23}$$

Above equation is solved by performing several iterations. Because, $\bar{F}4$ is the function of $\left\{\frac{\partial\bar{p}}{\partial\bar{\dot{h}}}\right\}$. Iterations are continued until convergence on pressure derivative with respect to the $\bar{\dot{h}}$ is obtained. Integration on air pressure derivative is performed on thrust bearing area as follows.

$$\begin{aligned}\bar{C}&=\frac{\partial\bar{F}}{\partial\bar{\dot{h}}}\\&=\sum_{e=1}^{n_e}\left\{\int_{-1}^{1}\int_{-1}^{1}\left(\sum_{i=1}^{n_l=4}\frac{\partial\bar{p}_i}{\partial\bar{\dot{h}}}N_i\right)|\bar{J}|d\xi d\eta\right\}+\sum_{i=1}^{i=n_p}\bar{A}_{oc}\frac{\partial\bar{p}_{oc}}{\partial\bar{\dot{h}}}\end{aligned} \tag{24}$$

4.4. Boundary conditions

The boundary conditions used to obtain solution as follows

1. All the nodes lying on the boundary have been assigned atmospheric pressure.
2. The nodes lying on the pocket boundary have equal pressure.
3. Air flow rate from restrictor must be equal to air input to the bearing system.

4.5. Newton Raphson iterative method

Newton Raphson method has a very fast convergence than any other numerical methods. The application of Newton Raphson method is quite involved in case of compressible Reynolds equation because Computation of derivatives of air fluidity matrix is difficult. The Newton Raphson method in aerostatic bearing has been applied by using following expression.

$$[\bar{p}_0]_{k+1}=[\bar{p}_0]_k-([\bar{F}4]_k)^{-1}\left(\{\bar{Q}\}+\bar{\dot{h}}[\bar{R}_t]-[\bar{F}1]\{\bar{p}_k\}\right) \tag{25}$$

k is the iteration number and iteration are performed until the desired performance criteria is satisfied. In the present work following convergence criteria has been used.

$$tol = 0.001 > max\left(\frac{\{\bar{p}_0\}_{k+1} - \{\bar{p}_0\}_k}{\{\bar{p}_0\}_k}\right) \quad (26)$$

5. Solution Procedure

Non-linear iterative finite element method is used to compute bearing performance parameters as described in earlier sections. The computation of air film stiffness and damping coefficient also involves an iterative procedure which makes whole numerical computation a cumbersome and time-consuming task. Therefore, in the present case an efficient finite element scheme is developed to compute the bearing performance parameters. The developed finite element matrixes are also function of air film pressure which makes iteration process sensitive to initial values of pressure. Therefore, a weight factor according to following equation is used to update the value of pressure.

$$\{\overline{\boldsymbol{p}}\}_{New} = (1 - w)\{\overline{\boldsymbol{p}}\}_{Old} + w\{\overline{\boldsymbol{p}}\}_{Computed} \quad (27)$$

To obtain the value of non-dimensional air film pressure, a solution scheme has been presented in figure 3. The complete solution procedure has been explained in the following simplified steps.

1. Aerostatic thrust bearing domain has been discretized by using four nodded isoparametric elements.
2. Values of air film pressure, air film pressure derivative with respect to $\bar{\bar{h}}$ and air film pressure derivative with respect to $\bar{h}$ are initialized.
3. Gauss points are generated in the elements and the elemental fluidity matrix $\overline{\boldsymbol{F}}1$, $\overline{\boldsymbol{F}}2$, $\overline{\boldsymbol{F}}3$, $\overline{\boldsymbol{F}}4$, $\overline{\boldsymbol{Q}}$ and $\bar{R}_t$ are computed.
4. Compute global linearized system of equations.
5. Global boundary conditions have been applied onto assemble global system of equations.

6. Apply Newton Raphson method and Compute air film pressure matrix by solving the system of equation.
7. Iterations are performed until the convergence is achieved in air film pressure and derivatives of pressure.
8. Check the convergence criteria of pressure otherwise repeat step number 3 to 7.
9. Compute stiffness and damping coefficient by using the derivative of air film pressure derivative with respect to $\bar{\dot{h}}$ and air film pressure derivative with respect to $\bar{h}$
10. Once all the convergence criteria in air film stiffness and damping are satisfied, program finalizes the solution.

6. Results and discussion

Four different types of recess shape having same ratio of $\frac{\bar{A}_b}{\bar{A}_{oc}} = 4$ have been chosen for the analysis as shown in figure 2. On the basis of above formulation, a MATLAB program has been developed to simulate the performance of bearing. To verify the present solution algorithm, the results is numerically verified with the available analytical results as shown in figure 4. The results have been validated with available analytical results for the circular thrust pad bearing having circular recess shape[28]. As shown in figure 4, results of numerical methods are reasonably good in agreement with available analytical results in literature. As the present work suggest a method to compute air film stiffness and damping coefficient of gas bearing therefore, results of Finite Element Method are compared with the results of perturbation method. As shown in Table 1, a good agreement has been observed in the value of air film stiffness and damping coefficient. The maximum difference in the value of stiffness is 0.23 % and whereas for the damping it is 0.26 %. In the figures 5-13, results are presented for the following operating and geometric parameters as represented in table 2. In order to depict the accuracy of the program and more physical insight, three-dimensional pressure distribution plots are presented through figures (5-8). The three-dimensional contour plots show that the tilt significantly changes the pressure distribution on the bearing and maximum change is observed for the circular recess thrust bearing. The geometric and operating parameters as indicated in Table 2 has been judiciously chosen on the basis of available literature.

Table 1: Comparison of bearing dynamic performance parameter by using perturbation and direct FEM approach

Restrictor design parameter	Performance parameter	Perturbation Method	Present FEM approach	% Difference
$\bar{C}_{s2} = 0.5$	S	1.6176	1.6194	-0.12
	C	1.9269	1.9219	0.26
$\bar{C}_{s2} = 1$	S	0.9935	0.9923	0.12
	C	1.2991	1.2958	0.25
$\bar{C}_{s2} = 1.4$	S	0.6552	0.6538	0.23
	C	1.0488	1.0507	-0.18

Table 2: Bearing geometric and operating parameters[4, 9, 27, 29]

S. No.	**Bearing Parameters**	**Values chosen**
1	Bearing diameter (D)	50 mm
2	Clearance (c)	10 µm
3	Orifice design parameter ($\bar{C}_{s2}$)	0.5-1.5
4	Tilt parameter (λ)	0-0.8
5	Ambient pressure (P_a)	1.01×10^5 Pa
6	Supplied pressure (P_s)	6 P_a
7	Air viscosity (η)	1.79756×10^{-5} N·s/m^2
8	Air density (ρ)	1.204 Kg/m^3
9	Air Temperature (T)	293 K
10	Acceleration of gravity (g)	9.8 m/s^2
11	Isentropic expansion index of air (κ)	1.4
12	Gas constant (R_g)	29.253s^2/(Kg·K)

6.1. Pocket Pressure$(\bar{p}_{oc})$

The variation of pocket pressure with orifice design parameter $(\bar{C}_{s2})$ is depicted in figure 9. As the value of orifice design parameter increases, the value of $\bar{p}_{oc}$ increases. The value of $\bar{p}_{oc}$ of tilted pad thrust bearing is less than parallel thrust bearing. For the selected value of $\bar{C}_{s2} = 1$, the percentage decrease in the value of $\bar{p}_{oc}$ is 7.9 %, 13.39 %, 18.23 % and 13.9 % for the recess shape circular, elliptical, annular and rectangular respectively. The circular recess shape offers higher value of pocket pressure as compare to other thrust bearing having recess shape elliptical, annular and rectangular. From the figure 9, following pattern has been observed in the value of pocket pressure $(\bar{p}_{oc})$.

$\bar{p}_{oc}|_{Circular} > \bar{p}|_{Elliptical} > \bar{p}_{oc}|_{Annular} > \bar{p}_{oc}|_{Rectangular}$ and $\bar{p}_{oc}|_{Parrallel} > \bar{p}_{oc}|_{Tilt}$

6.2. Load carrying capacity$(\bar{F}_0)$

Figure 10 depicts the variation of load carrying capacity$(\bar{F}_0)$ with orifice design parameter $(\bar{C}_{s2})$. A positive change in orifice design parameter results a positive change in the value of pockets pressure therefore, load carrying capacity of bearing increases with increasing value of orifice designer parameter. The value of $\bar{F}_0$ of parallel thrust pad bearing is higher than that of tilted pad thrust bearing. For the chosen value of $\bar{C}_{s2} = 1$, the percentage decrease in the value of $\bar{F}_0$ is 4.90 %, 11.64 %, 17.08 % and 12.34 % for the circular, elliptical, annular and rectangular recess shape respectively. As the load carrying capacity is an important parameter for bearing design and designer have the choice of recess shape, the designer can choose the bearing on the basis of following pattern.

$\bar{F}_0\big|_{Circular} > \bar{F}_0\big|_{Elliptical} > \bar{F}_0\big|_{Annular} > \bar{F}_0\big|_{Rectangular}$ and $\bar{F}_0\big|_{Parrallel} > \bar{F}_0\big|_{Tilt}$

6.3. Air flow rate$(\bar{Q}_0)$

Figure 11 represents the variation of air flow rate with increasing value of orifice designer parameter. The value of air flow rate increases with increasing value of orifice restrictor design parameter and air flow rate of tilted pad thrust bearing is significantly more than that of thrust bearing of parallel pad. For the selected value of $\bar{C}_{s2} = 1$, percentage increase in the value of $\bar{Q}_0$ is 24.04 %, 29.43 %, 11.43 % and 27.44 % for the recess shape circular, elliptical, annular and rectangular respectively. The thrust bearing that has annular recess shape in tilt operation has the maximum value air flow rate while the thrust bearing that has the circular recess shape has the minimum value of air flow rate. From the figure 11, the following general pattern is observed in the value air flow rate of tilted pad thrust bearing.

$\bar{Q}_0|_{Circular} < \bar{Q}_0|_{Elliptical} > \bar{Q}_0|_{Rectangular} > \bar{Q}_0|_{Annular}$ and $\bar{Q}_0|_{Parrallel} > \bar{Q}_0|_{Tilt}$

6.4. Air film Stiffness coefficient($\bar{S}$)

Figure 12 depicts the variation of the air film stiffness coefficient ($\bar{S}$) with the tilt parameter for various recessed shape thrust bearings. As the value of orifice parameter increases, the value of air film stiffness coefficient increases. For the chosen value of $\bar{C}_{s2} = 0.9$, the percentage decrease in the value of $\bar{S}$ is 22.0 %, 6.5 %, 19.58 % and 2.37 % for the recess shape circular, elliptical, annular and rectangular respectively. The tilt in thrust pad results in a significant reduction in the value of air film stiffness coefficient of aerostatic thrust pad bearing. From the designer point of view stiffness is an important parameter to absorb bearing vibration and designer has the choice of recess shape, designer can choose bearing from following pattern.

$\bar{S}|_{Circular} < \bar{S}|_{Elliptical} > \bar{S}|_{Rectangular} > \bar{S}|_{Annular}$ and $\bar{S}|_{tilted} < \bar{S}|_{without\ Tilt}$

6.5. Air film Damping coefficient ($\bar{C}$)

Figure 13 depicts the variation of the air film damping coefficient ($\bar{C}$) with the tilt parameter for various recessed shape thrust bearings. The value of damping coefficient increases with an increase in the value of tilt parameter. For the chosen value of $\bar{C}_{s2} = 0.7$, the percentage increase in the value of $\bar{C}$ is 119.1 %, 65.75 %, 13.27 % and 56.96 % for the recess shape circular, elliptical, annular and rectangular respectively. The value of air film damping coefficient monotonically decreases with an increase in the value of orifice design parameter.

As damping is an important capability to absorb vibration in bearing. Higher value of damping means good ability to absorb vibration and lower value of air film damping coefficient means bearing has lower ability to absorb fluctuations.

$\bar{C}|_{tilted} > \bar{C}|_{without\ Tilt}$ and $\bar{C}|_{Circular} > \bar{C}|_{Elliptical} > \bar{C}|_{Rectangular} > \bar{C}|_{Annular}$

7. Conclusions

A novel technique based on finite element formulation is presented to calculate the static and dynamic performance parameters of aerostatic thrust bearing. Further, the effect of tilt parameter and recess shape has been presented aerostatic thrust bearing. On the basis of numerical simulated results, following important conclusions may be drawn.

1. The static and dynamic performance of an aerostatic tilted thrust pad bearing is significantly affected with a change in the value of tilt parameter and the shape of the recess.
2. Load carrying capacity of tilted pad aerostatic thrust bearing is less than the bearing without tilt. Whereas the value air film stiffness coefficient of tilted thrust pad is more as compare to parallel thrust pad bearing.

   $\bar{F}_0|_{Tilt} < \bar{F}_0|_{Without\ Tilt}$, $\bar{S}|_{Tilt} < \bar{S}|_{Without\ Tilt}$,
3. Air film Stiffness coefficient of the bearing significantly changes with tilt parameter and recess shape of bearing. Maximum percentage change in the value of tilt is up to 22.8% for circular thrust pad bearing. Whereas air film damping of bearing changes also significantly changes with tilt and recess shape, this change is upto 120% for the thrust pad bearing having circular recess shape.

   $\bar{C}|_{Tilt} > \bar{C}|_{Without\ Tilt}$
4. The finite element based numerical method is quite useful and accurate for the computation of air film stiffness and damping coefficient.
5. It is expected that simulated result will be quite useful for the aerostatic bearing

designers.

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