$\eta-\pi^0$ MIXING AND $\psi' \to \psi\pi^0$ DECAY

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The $\psi' \to \psi\pi^0$ decay rate is studied in a chiral symmetry breaking scheme by including effects from $\pi^0-\eta$ mixing only. The result obtained is in very good agreement with the experiment.

It has recently been pointed out by Langacker [1] that the large experimental branching ratio R for the isospin violating decay $\psi' \to \psi\pi^0$ given by [2]

$$R = B(\psi' \to \psi\pi^0)/B(\psi' \to \psi\eta) = (39 \pm 10) \times 10^{-3} \text{ or } (60 \pm 30) \times 10^{-3} \tag{1}$$

can be explained in a simple symmetry breaking model by assuming that the violation takes place via annihilation of a $c\bar{c}$ pair into $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ analogues. In this letter we show that such a large branching ratio may be also accounted for in a chiral symmetry framework by modifying the PCAC relations for π and η to include effects of $\langle\eta|\pi\rangle$ overlap and suitably evaluating the time ordered product of axial vector currents using the standard spectral representation.

To start with, we write down the $\langle\eta|\pi\rangle$ overlap as

$$\langle\eta|\pi\rangle = -\mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{-\mathrm{i}kx}(k^2+m_\eta^2)(k^2+m_\pi^2)\langle 0|T\{\phi_\eta(x)\phi_\pi(0)\}|0\rangle\,, \tag{2}$$

where the fields ϕ_η and ϕ_π are defined by [3]

$$\partial_\mu A_\mu^{(3)} = f_\pi(m_\pi^2\phi_\pi + \langle\eta|\pi\rangle\phi_\eta)\,,\qquad \partial_\nu A_\nu^{(8)} = f_\eta(m_\eta^2\phi_\eta + \langle\eta|\pi\rangle\phi_\pi)\,, \tag{3}$$

neglecting $\pi-\eta'$ and $\eta'-\eta$ mixings. The $\eta-\pi$ mixing angle θ is related to $\langle\eta|\pi\rangle$ as

$$\theta = \langle\eta|\pi\rangle/(m_\eta^2 - m_\pi^2)\,. \tag{4}$$

By substituting eq. (3) in eq. (2) we get, for small k^2,

$$\mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{-\mathrm{i}kx}\langle 0|T\{\partial_\mu A_\mu^{(8)}(x)\,\partial_\nu A_\nu^{(3)}(0)\}|0\rangle = \langle\eta|\pi\rangle\left(-1 + \frac{k^2+m_\pi^2}{m_\pi^2} + \frac{k^2+m_\eta^2}{m_\eta^2}\right)\frac{f_\pi f_\eta m_\pi^2 m_\eta^2}{(k^2+m_\pi^2)(k^2+m_\eta^2)}\,, \tag{5}$$

in which terms involving second order in $\langle\eta|\pi\rangle$ are neglected.

Applying now the standard reduction techniques, one can express eq. (5) as [‡1]

[‡1] We have taken a soft meson limit $k_\mu \to 0$ to evaluate the second term on the rhs of eq. (6). For the first term, however, we shall make a low energy approximation viz. $E_\pi \approx \mathrm{i}m_\pi$ and shall use, in what follows, a weaker limit $k^2 \to 0$.

$$k_\mu k_\nu \Delta_{\mu\nu} = \langle\eta|\pi\rangle\left(1+\frac{k^2}{m_\pi^2}+\frac{k^2}{m_\eta^2}\right)\frac{f_\pi f_\eta m_\pi^2 m_\eta^2}{(k^2+m_\pi^2)(k^2+m_\eta^2)} + \left(\frac{1}{3}\right)^{1/2}\frac{m_\mathrm{d}-m_\mathrm{u}}{m_\mathrm{d}+m_\mathrm{u}} f_\pi^2 m_\pi^2 , \tag{6}$$

where

$$\Delta_{\mu\nu} = \mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{-\mathrm{i}kx}\langle 0|T\{A_\mu^{(8)}(x)A_\nu^{(3)}(0)\}|0\rangle . \tag{7}$$

We next use the standard spectral representation [4] for $\Delta_{\mu\nu}$ to evaluate [‡2] the lhs of eq. (6). We get, after some algebra,

$$k_\mu k_\nu \Delta_{\mu\nu} = k^2 \int \mathrm{d}m^2\, \frac{\zeta^{(8,3)}(m^2)}{m^2} + k^4 f_\pi f_\eta\left(\frac{ab}{k^2+m_\pi^2}+\frac{cd}{k^2+m_\eta^2}\right) - k^2\langle\eta|\pi\rangle f_\pi f_\eta\left(\frac{a^2+b^2}{k^2+m_\pi^2}+\frac{c^2+d^2}{k^2+m_\eta^2}\right)$$
$$+ \text{ST (Schwinger terms)} , \tag{8}$$

where

$$\mathrm{ST} \approx m_\pi m_\eta \left\{\int \mathrm{d}m^2\, \frac{\zeta^{(8,3)}(m^2)}{m^2} + f_\pi f_\eta\left[(ab+cd)+\left(\frac{a^2+b^2}{m_\pi^2}+\frac{c^2+d^2}{m_\eta^2}\right)\langle\eta|\pi\rangle\right]\right\}, \tag{9}$$

and

$$a = \langle 0|\phi_\pi|\pi\rangle , \quad b = \langle 0|\phi_\pi|\eta\rangle , \quad c = \langle 0|\phi_\eta|\pi\rangle , \quad d = \langle 0|\phi_\eta|\eta\rangle . \tag{10}$$

To leading order, the matrix elements a, b, c and d are now evaluated [5] by taking eqs. (3) between $\langle 0|$ and $|\pi\rangle$. or $|\eta\rangle$ state as the case may be:

$$a = K[x m_\eta^2 f_\eta/f_\pi - (1/\sqrt{3})\langle\eta|\pi\rangle] , \quad b = K[-x\langle\eta|\pi\rangle f_\eta/f_\pi + (1/\sqrt{3})m_\pi^2] ,$$
$$c = K[(1/\sqrt{3})m_\eta^2 - \tfrac{1}{3}y\langle\eta|\pi\rangle f_\eta/f_\pi] , \quad d = K[-(1/\sqrt{3})\langle\eta|\pi\rangle + \tfrac{1}{3}y m_\pi^2 f_\pi/f_\eta] , \tag{11}$$

where k is a constant and x and y are:

$$x = -(m_\mathrm{d}+m_\mathrm{u})/(m_\mathrm{d}-m_\mathrm{u}) , \quad y = -(4m_\mathrm{s}+m_\mathrm{d}+m_\mathrm{u})/(m_\mathrm{d}-m_\mathrm{u}) . \tag{12}$$

Substituting eqs. (9)–(12) in eq. (8) and neglecting octet–triplet axial vector mixing, we obtain

$$\langle\eta|\pi\rangle\left(1+\frac{k^2}{m_\pi^2}+\frac{k^2}{m_\eta^2}\right)\frac{m_\pi^2 m_\eta^2}{(k^2+m_\pi^2)(k^2+m_\eta^2)} = k^4\left(\frac{ab}{k^2+m_\pi^2}+\frac{cd}{k^2+m_\eta^2}\right) - \langle\eta|\pi\rangle k^2\left(\frac{a^2+b^2}{k^2+m_\pi^2}+\frac{c^2+d^2}{k^2+m_\eta^2}\right)$$
$$-\left(\frac{1}{3}\right)^{1/2}\frac{m_\pi^2}{x} + m_\pi m_\eta\left[(ab+cd)+\left(\frac{a^2+b^2}{m_\pi^2}+\frac{c^2+d^2}{m_\eta^2}\right)\langle\eta|\pi\rangle\right] \tag{13}$$

where $f_\pi = f_\eta$ has been assumed [5].
Since eq. (13) is valid for all small k^2, one has at $k^2 = 0$ the "smoothness" relation

$$\langle\eta|\pi\rangle = m_\pi m_\eta\left[(ab+cd)+\frac{a^2+b^2}{m_\pi^2}+\frac{c^2+d^2}{m_\eta^2}\Big)\langle\eta|\pi\rangle\right] - \left(\frac{1}{3}\right)^{1/2}\frac{m_\pi^2}{x} , \tag{14}$$

[‡2] The use of the spectral representation for $\Delta_{\mu\nu}$ to evaluate $k_\mu k_\nu \Delta_{\mu\nu}$ makes the spirit of our paper different from that adopted in ref. [3].

which gives the following estimates for $\langle\eta|\pi\rangle$ overlap and θ;

$$\langle\eta|\pi\rangle = -0.016\ (\mathrm{GeV})^2 \qquad \theta = -5.7 \times 10^{-2} \tag{15}$$

for ‡3 $x = -3.9$ and $y = -180.5$. It may be noted that the value of θ as obtained here ‡4,5 is considerably larger than what so far existed in the literature with [9] or without [10] the inclusion of the effect of η'.
Eq. (15) leads to

$$R = B(\Psi' \to \Psi\pi)/B(\Psi' \to \Psi\eta) \approx 15 \times \theta^2 = 48.7 \times 10^{-3} \tag{16}$$

which is in very good agreement with its experimental [2] value of either $(39 \pm 10) \times 10^{-3}$ or $(60 \pm 30) \times 10^{-3}$.

Finally, a comment on the $\pi^\pm - \pi^0$ mass difference from the $\eta-\pi$ mixing obtained in eq. (15) seems worthwhile here. This is because the $\eta-\pi$ mixing term in the mass matrix reduces the π^0 mass relative to $\pi^\pm$ and one has, to leading order [11],

$$\Delta m_\pi = m_{\pi^\pm} - m_{\pi^0} = (1/2\sqrt{3})\,(\theta/x)\,m_{\pi^\pm}\ . \tag{17}$$

It is evident from the above equation that a significant contribution to Δm_π is expected from the $\eta-\pi$ mixing that increases with the mixing parameter θ. Indeed for the value of θ in eq. (15), Δm_π turns out to be 0.5–0.6 MeV which is about 12% of the observed mass difference $(\Delta m_\pi)_{\mathrm{exp}} = 4.6$ MeV. However, as pointed out by Gross et al. [11], since the isospin violating electromagnetic contributions account for most of the pion mass splitting and since corrections to PCAC are of the order [12] of 15% of $(\Delta m_\pi)_{\mathrm{exp}}$, *any* contribution from a quark mass difference can only increase the discrepancy with experiment ‡6,7. It may be noted here that an estimate of the electromagnetic contribution to Δm_π may be given by $(\Delta m_\pi) \approx 6.1 \pm 0.8$ MeV by following the PCAC analysis of Das et al. [13] and using the present experimental determination of the ρ-coupling constant.

We are investigating [14] the reactions $\pi^- p \to n\eta$, $\pi^+ n \to \eta p$ and the $\eta, \eta' \to 3\pi$ decay to estimate the $\eta - \pi^0$ mixing. Details of these as well as effects of $\eta'-\pi^0$ mixing on our results will be communicated at a later date.

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‡3 The values of x and y taken here are within the errors of the estimates made by Dominguez [6].

‡4 θ is mildly sensitive to the changes in the values of x and y. If one takes [7] $x = -3.5$ and $y = -183.5$, θ turns out to be $\theta = -4 \times 10^{-2}$ yielding $R = 30 \times 10^{-3}$, in agreement with the experimental value of Peck, ref. [2].

‡5 We have recently obtained [8] $\theta = -4.6 \times 10^{-2}$ by making use of Weinberg's first spectral function sum rule.

‡6 See Gross et al., ref. [11], for a detailed discussion on this point.

‡7 Unless, of course, the sign of θ is different. It may be mentioned in this connection that, by including the effect of η', Oneda et al. [9] had obtained two distinct values of θ that differed in sign. Moreover, one value of θ there is about the same order of magnitude (but off by a factor of 3) as obtained by us in eq. (15) and another close in magnitude to the one obtained by Okubo and Sakita [10] without considering the η' effects.

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