Functional integral treatment of some quantum nondemolition systems

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Abstract. In the scheme of a quantum nondemolition (QND) measurement, an observable is measured without perturbing its evolution. In the context of studies of decoherence in quantum computing, we examine the 'open' quantum system of a two-level atom, or equivalently, a spin-1/2 system, in interaction with quantum reservoirs of either oscillators or spins, under the QND condition of the Hamiltonian of the system commuting with the system-reservoir interaction. For completeness, we also examine the well-known non-QND spin-Bose problem. For all these many-body systems, we use the methods of functional integration to work out the propagators. The propagators for the QND Hamiltonians are shown to be analogous to the squeezing and rotation operators, respectively, for the two kinds of baths considered. Squeezing and rotation being both phase space area-preserving canonical transformations, this brings out an interesting connection between the energy-preserving QND Hamiltonians and the homogeneous linear canonical transformations.

PACS numbers: 03.65.Db, 03.65.Yz, 42.50.Ct

1. Introduction

In the scheme of a quantum nondemolition (QND) measurement, an observable is measured without perturbing its free motion. Such a scheme was originally introduced in the context of the detection of gravitational waves [1]. It was to counter the quantum mechanical unpredictability that in general would disturb the system being measured. The dynamical evolution of a system immediately following a measurement limits the class of observables that may be measured repeatedly with arbitrary precision, with the influence of the measurement apparatus on the system being confined strictly to the conjugate observables. Observables having this feature are called QND or back-action evasion observables [2, 3, 4]. In addition to its relevance in ultrasensitive measurements, a QND scheme provides a way to prepare quantum mechanical states which may otherwise be difficult to create, such as Fock states with a specific number of particles. One of the original proposals for a quantum optical QND scheme was that involving the Kerr medium [5], which changes its refractive index as a function of the number of photons in the 'signal' pump laser. The advent of experimental methods for producing Bose-Einstein condensation (BEC) enables us to make progress in the matter-wave analogue of the optical QND experiments. In the context of research into BEC, QND schemes with atoms are particularly valuable, for instance, in engineering entangled states or Schrödinger's cat states. A state preparation with BEC has recently been performed in the form of squeezed state creation in an optical lattice [6]. In a different context, it has been shown that the accuracy of atomic interferometry can be improved by using QND measurements of the atomic populations at the inputs to the interferometer [7].

No system of interest, except the entire universe, can be thought of as an isolated system – all subsets of the universe are in fact 'open' systems, each surrounded by a larger system constituting its environment. The theory of open quantum systems provides a natural route for reconciliation of dissipation and decoherence with the process of quantization. In this picture, friction or damping comes about by the transfer of energy from the 'small' system (the system of interest) to the 'large' environment. The energy, once transferred, disappears into the environment and is not given back within any time of physical relevance. Ford, Kac and Mazur [8] suggested the first microscopic model describing dissipative effects in which the system was assumed to be coupled to a reservoir of an infinite number of harmonic oscillators. Interest in quantum dissipation, using the system-environment approach, was intensified by the works of Caldeira and Leggett [9], and Zurek [10] among others. The path-integral approach, developed by Feynman and Vernon [11], was used by Caldeira and Leggett [9], and the reduced dynamics of the system of interest was followed taking into account the influence of its environment, quantified by the influence functional. In the model of the fluctuating or "Brownian" motion of a quantum particle studied by Caldeira and Leggett [9], the coordinate of the particle was coupled linearly to the harmonic oscillator reservoir, and it was also assumed that the system and the environment were initially factorized. The treatment of the quantum Brownian motion has since been generalized to the physically reasonable initial condition of a mixed state of the system and its environment by Hakim and Ambegaokar [12], Smith and Caldeira [13], Grabert, Schramm and Ingold [14], and by us for the case of a system in a Stern-Gerlach potential [15], and also for the quantum Brownian motion with nonlinear system-environment couplings [16].

An open system Hamiltonian is of the QND type if the Hamiltonian H_S of the system commutes with the Hamiltonian H_{SR} describing the system-reservoir interaction, i.e., H_{SR} is a constant of motion generated by H_S. Interestingly, such a system may still undergo decoherence or dephasing without any dissipation of energy [17, 18].

In this paper, we study such QND 'open system' Hamiltonians of particular interest in the context of decoherence in quantum computing, and obtain the propagators of the composite systems explicitly using path integral methods, for two different models of the environment. The aim is to shed some light on the problem of QND measurement schemes. Can one draw upon any familiar symmetries to connect with the time-evolution operation of these QND systems of immense physical importance?

We take our system to be a two-level atom, or equivalently, a spin-1/2 system. We consider two types of environment, describable as baths of either oscillators or spins. One cannot in general map a spin-bath to an oscillator-bath (or vice versa); they constitute distinct universality classes of quantum environment [19]. The first case of oscillator-bath models (originated by Feynman and Vernon [11]) describes delocalized environmental modes. For the spin-bath, on the other hand, the finite Hilbert space of each spin makes it appropriate for describing the low-energy dynamics of a set of localized environmental modes. A difficulty associated with handling path integrals for spins comes from the discrete matrix nature of the spin-Hamiltonians. This difficulty is overcome by bosonizing the Hamiltonian by representing the spin angular momentum operators in terms of boson operators following Schwinger's theory of angular momentum [20].

We then use the Bargmann representation [21] for all the boson operators. The Schrödinger representation of quantum states diagonalizes the position operator, expressing pure states as wave functions, whereas the Bargmann representation diagonalizes the creation operator $b^\dagger$, and expresses each state vector $|\psi\rangle$ in the Hilbert state $\mathcal{H}$ as an entire analytic function $f(\alpha)$ of a complex variable α. The association $|\psi\rangle \longrightarrow f(\alpha)$ can be written conveniently in terms of the normalized coherent states $|\alpha\rangle$ which are the right eigenstates of the annihilation operator b:

$$\begin{aligned} b|\alpha\rangle \quad &= \alpha|\alpha\rangle, \\ \langle\alpha'|\alpha\rangle &= \exp\left(-\frac{1}{2}|\alpha'|^2 - \frac{1}{2}|\alpha|^2 + \alpha'^*\alpha\right), \end{aligned}$$

giving

$$f(\alpha) = e^{-|\alpha|^2/2}\ \langle\alpha^*|\psi\rangle.$$

We obtain the explicit propagators for these many-body systems from those of the expanded bosonized forms by appropriate projection.

The propagators for the QND Hamiltonians with an oscillator bath and a spin bath are shown to be analogous to the squeezing and rotation operators, respectively, which are both phase space area-preserving canonical transformations. This suggests an interesting connection between the energy-preserving QND Hamiltonians and the homogeneous linear canonical transformations, which would need further systematic probing.

The plan of the paper is as follows. In section 2 we take up the case of a QND-type of open system Hamiltonian where the bath is a bosonic one of harmonic oscillators. In section 2.1 we consider a case, which is a variant of the previous one, wherein we include an external mode in resonance with the atomic transition and obtain its propagator. In section 2.2 we discuss the non-QND variant of the Hamiltonian which usually occurs in the literature in discussions of the spin-Bose problem [22, 23]. In section 3 we treat the case of a QND-type of open system Hamiltonian where the bath is composed of two-level systems or spins. The structure of the propagators in the two cases of the oscillator and spin baths is discussed in section 4, and in section 5 we present our conclusions.

2. Bath of harmonic oscillators

We first take the case where the system is a two-level atom interacting with a bosonic bath of harmonic oscillators with a QND type of coupling. Such a model has been studied [24, 25, 26] in the context of the influence of decoherence in quantum computation. The total system evolves under the Hamiltonian,

$$\begin{aligned} H_1 &= H_S + H_R + H_{SR} \\ &= \frac{\hbar\omega}{2}\sigma_z + \sum_{k=1}^{M} \hbar\omega_k b_k^\dagger b_k + \left(\frac{\hbar\omega}{2}\right) \sum_{k=1}^{M} g_k (b_k + b_k^\dagger)\sigma_z. \end{aligned} \tag{1}$$

Here H_S, H_R and H_{SR} stand for the Hamiltonians of the system, reservoir, and system-reservoir interaction, respectively. We have made use of the equivalence of a two-level atom and a spin-1/2 system, σ_x, σ_z denote the standard Pauli spin matrices and are related to the spin-flipping (or atomic raising and lowering) operators S_+ and S_-: $\sigma_x = S_+ + S_-$, $\sigma_z = 2S_+S_- - 1$. In (1) $b_k^\dagger, b_k$ denote the Bose creation and annihilation operators for the M oscillators of frequency ω_k representing the reservoir, g_k stands for the coupling constant (assumed real) for the interaction of the field with the spin. Since $[H_S, H_{SR}] = 0$, the Hamiltonian (1) is of QND type.

The explicit propagator $\exp(-\frac{iHt}{\hbar})$ for the Hamiltonian (1) is obtained by using functional integration and bosonization [22, 27]. In order to express the spin angular momentum operators in terms of boson operators, we employ Schwinger's theory of angular momentum [20] by which any angular momentum can be represented in terms of a pair of boson operators with the usual commutation rules. The spin operators σ_z and σ_x can be written in terms of the boson operators a_β, $a_\beta^\dagger$ and a_γ, $a_\gamma^\dagger$ as

$$\begin{aligned} \sigma_z &= a_\gamma^\dagger a_\gamma - a_\beta^\dagger a_\beta, \\ \sigma_x &= a_\gamma^\dagger a_\beta + a_\beta^\dagger a_\gamma. \end{aligned}$$

In the Bargmann representation [21] the actions of b and $b^\dagger$ are

$$\begin{aligned} b^\dagger f(\alpha) &= \alpha^* f(\alpha), \\ b f(\alpha) &= \frac{df(\alpha)}{d\alpha^*}, \end{aligned} \tag{2}$$

where $|\alpha\rangle$ is the normalized coherent state. The spin operator becomes

$$\sigma_z \longrightarrow \left(\gamma^* \frac{\partial}{\partial \gamma^*} - \beta^* \frac{\partial}{\partial \beta^*}\right). \tag{3}$$

Here the variable β^* is associated with the spin-down state and the variable γ^* with the spin-up state.

The bosonized form of the Hamiltonian (1) is

$$\begin{aligned} H_{B_1} = \frac{\hbar\omega}{2} &\left(\gamma^* \frac{\partial}{\partial \gamma^*} - \beta^* \frac{\partial}{\partial \beta^*}\right) + \sum_{k=1}^{M} \hbar\omega_k \alpha_k^* \frac{\partial}{\partial \alpha_k^*} \\ &+ \frac{\hbar\omega}{2} \sum_{k=1}^{M} g_k \left(\alpha_k^* + \frac{\partial}{\partial \alpha_k^*}\right) \left(\gamma^* \frac{\partial}{\partial \gamma^*} - \beta^* \frac{\partial}{\partial \beta^*}\right). \end{aligned} \tag{4}$$

Here α_k^*, $\frac{\partial}{\partial \alpha_k^*}$ are the Bargmann representations for $b_k^\dagger$ and b_k, respectively. A particular solution of the Schrödinger equation for the bosonized Hamiltonian (4) is

$$U_1 = U_{00}\beta^*\beta' + U_{01}\beta^*\gamma' + U_{10}\gamma^*\beta' + U_{11}\gamma^*\gamma', \tag{5}$$

where the amplitude U_{ij} are functions of time as well as the coherent state variables associated with the boson oscillators, with the initial condition

$$U_{ij}(t=0) = \exp\left\{\sum_{k=1}^{M} \alpha_k^* \alpha_k'\right\} \delta_{ij} \qquad (i,j=0,1). \tag{6}$$

The initial state for the expanded propagator associated with the bosonized Hamiltonian (5) is

$$U(t=0) = \exp\left\{\sum_{k=1}^{M} \alpha_k^* \alpha_k'\right\} \exp\left\{\beta^*\beta' + \gamma^*\gamma'\right\}. \tag{7}$$

If the Hamiltonian is in the normal form given by $H(\alpha^*, \frac{\partial}{\partial \alpha^*}, t)$, the associated propagator is given as a path integral over coherent state variables as [28]

$$U(\alpha^*, t; \alpha', 0) = \int \mathbf{D}\{\alpha\} \exp\left\{\sum_{0\le\tau<t} \alpha^*(\tau+)\alpha(\tau) - \frac{i}{\hbar}\int_0^t d\tau H\left(\alpha^*(\tau+), \alpha(\tau), \tau\right)\right\}. (8)$$

Here $\sum_{0\le\tau<t} \alpha^*(\tau+)\alpha(\tau)$ stands for $\sum_{j=0}^{N-1} \alpha^*(\tau_{j+1})\alpha(\tau_j)$ in the subdivision of the internal $[0,t]$, i.e., where τ stands for τ_j, $\tau+$ stands for the next point τ_{j+1} in the subdivision. Also, in the subdivision scheme,

$$\int_0^t d\tau H\left(\alpha^*(\tau+), \alpha(\tau), \tau\right) = \sum_{j=0}^{N-1} H\left(\alpha^*(\tau_{j+1}), \alpha(\tau_j), \tau_j\right)\Delta\tau_j.$$

Here the path differential in (8) is

$$\mathbf{D}^2\{\alpha\} = \prod_{0<\tau<t} D^2\alpha(\tau), \tag{9}$$

where the weighted differential is

$$D^2\alpha(\tau) = \frac{1}{\pi}\exp\left(-|\alpha(\tau)|^2\right) d^2\alpha(\tau). \tag{10}$$

Using (8), the propagator for the bosonized Hamiltonian (4) is

$$\begin{aligned} u_1(\boldsymbol{\alpha^*}, \beta^*, \gamma^*, t; \boldsymbol{\alpha'}, \beta', \gamma', 0) = &\int \mathbf{D}^2\{\boldsymbol{\alpha}\}\mathbf{D}^2\{\beta\}\mathbf{D}^2\{\gamma\} \\ &\times \exp\left\{\sum_{0\le\tau<t}\left[\sum_{k=1}^{M}\alpha_k^*(\tau+)\alpha_k(\tau)\right.\right. \\ &\left. + \beta^*(\tau+)\beta(\tau) + \gamma^*(\tau+)\gamma(\tau)\right] \\ &- i\sum_{k=1}^{M}\int_0^t d\tau \omega_k \alpha_k^*(\tau+)\alpha_k(\tau) \\ &- i\frac{\omega}{2}\int_0^t d\tau\left[\gamma^*(\tau+)\gamma(\tau) - \beta^*(\tau+)\beta(\tau)\right] \\ &- i\frac{\omega}{2}\sum_{k=1}^{M}\int_0^t d\tau g_k\left[\alpha_k^*(\tau+) + \alpha_k(\tau)\right]\left[\gamma^*(\tau+)\gamma(\tau)\right. \\ &\left.\left. - \beta^*(\tau+)\beta(\tau)\right]\right\}. \end{aligned} \tag{11}$$

In Eq. (11) $\boldsymbol{\alpha}$ is a vector with components $\{\alpha_k\}$, and $\mathbf{D}^2\{\boldsymbol{\alpha}\} = \prod_{k=1}^{M} \mathbf{D}^2\{\alpha_k\}$.

Now we introduce a complex auxiliary field $f(\tau)$ to decouple the interaction term in (11) as

$$\exp\left(-\frac{i\omega}{2}\sum_{k=1}^{M}\int_0^t d\tau g_k\left[\alpha_k^*(\tau+)+\alpha_k(\tau)\right]\left[\gamma^*(\tau+)\gamma(\tau)-\beta^*(\tau+)\beta(\tau)\right]\right)$$
$$= \int \mathbf{D}^2\{f\}\exp\left[-i\sum_{k=1}^{M}\int_0^t d\tau f^*(\tau)g_k\left(\alpha_k^*(\tau+)+\alpha_k(\tau)\right)\right]$$
$$\times\exp\left[\int_0^t d\tau f(\tau)\frac{\omega}{2}\left(\gamma^*(\tau+)\gamma(\tau)-\beta^*(\tau+)\beta(\tau)\right)\right]. \tag{12}$$

Here we have used the δ-functional identify, [22]

$$\int \mathbf{D}^2\{x\}P[x^*(t)]\exp\left\{\int_0^t d\tau y(\tau)x(\tau)\right\} = P[y(t)], \tag{13}$$

where $\mathbf{D}^2\{x\}$ is the functional differential

$$\mathbf{D}^2\{x\} = \exp\left(-\int_0^t d\tau|x(\tau)|^2\right)\prod_{0\le\tau<t}\left(\frac{d\tau}{\pi}\right)d^2x(\tau), \tag{14}$$

and $\mathbf{P}[x^*(t)]$ is an explicit functional of x^* only. Using (12), the bosonized propagator (11) can be written as

$$u_1(\boldsymbol{\alpha^*},\beta^*,\gamma^*,t;\boldsymbol{\alpha'},\beta',\gamma',0) = \int \mathbf{D}^2\{f\}G_1(\boldsymbol{\alpha^*},t;\boldsymbol{\alpha'},0;[f^*])$$
$$\times N_1\left(\beta^*,\gamma^*,t;\beta',\gamma',0;[f]\right). \tag{15}$$

Here G_1 stands for the propagator for

$$H_{G_1} = \hbar\sum_{k=1}^{M}\left[\omega_k\alpha_k^*\frac{\partial}{\partial\alpha_k^*} + f^*(t)g_k\alpha_k^* + f^*(t)g_k\alpha_k\right], \tag{16}$$

N_1 is the propagator for

$$H_{N_1} = \frac{\hbar\omega}{2}\left(\gamma^*\frac{\partial}{\partial\gamma^*} - \beta^*\frac{\partial}{\partial\beta^*}\right) + \frac{i\hbar\omega}{2}f(t)\left(\gamma^*\frac{\partial}{\partial\gamma^*} - \beta^*\frac{\partial}{\partial\beta^*}\right). \tag{17}$$

These obey the Schrödinger equations $i\hbar\frac{\partial}{\partial t}G_1 = H_{G_1}G_1, i\hbar\frac{\partial}{\partial t}N_1 = H_{N1}N_1$ with the initial conditions

$$G_1(t=0) = \exp\left\{\sum_{k=1}^{M}\alpha_k^*\alpha_k'\right\},$$
$$N_1(t=0) = \exp\left\{\beta^*\beta' + \gamma^*\gamma'\right\}. \tag{18}$$

The propagator G_1 is given by

$$G_1 = \exp\left\{\sum_{k=1}^{M}\alpha_k^*\alpha_k' e^{-i\omega_k t} - \sum_{k=1}^{M}\left[i\alpha_k^* g_k\int_0^t d\tau f^*(\tau)e^{-i\omega_k(t-\tau)}\right.\right.$$
$$+ i\alpha_k' g_k\int_0^t d\tau e^{-i\omega_k\tau}f^*(\tau)$$
$$\left.\left.+ g_k^2\int_0^t d\tau\int_0^\tau d\tau' e^{-i\omega_k(\tau-\tau')}f^*(\tau)f^*(\tau')\right]\right\}. \tag{19}$$

The propagator N_1 is given by

$$N_1 = \exp\{Q_{00}\beta^*\beta' + Q_{01}\beta^*\gamma' + Q_{10}\gamma^*\beta' + Q_{11}\gamma^*\gamma'\}$$
$$= \sum_{l=0}^{\infty} \frac{1}{l!}\left[(\beta^*,\gamma^*)Q\begin{pmatrix}\beta'\\ \gamma'\end{pmatrix}\right]^l. \tag{20}$$

Here $Q(t)$ is given by

$$Q(t) = \exp\left(\frac{i\omega}{2}\sigma_z t - \frac{\omega}{2}\sigma_z \int_0^t d\tau f(\tau)\right), \tag{21}$$

with $Q_{ij}(0) = \delta_{ij}, Q(0) = I$.

Thus the propagator for the bosonized Hamiltonian (4) as given by (15) becomes

$$u_1 = \sum_{l=0}^{\infty} \int \mathbf{D}^2\{f\} G_1 \frac{1}{l!}\left[(\beta^*,\gamma^*)Q\begin{pmatrix}\beta'\\ \gamma'\end{pmatrix}\right]^l. \tag{22}$$

The propagator for the Hamiltonian (1) is obtained from (22) by taking the $l = 1$ term in the above equation. By making use of the δ-functional identity (13) the amplitudes of the propagator for the Hamiltonian (1) are obtained in matrix form as

$$u_1 = \begin{pmatrix} U_{00} & U_{01}\\ U_{10} & U_{11}\end{pmatrix} = \exp\left\{\sum_{k=1}^{M} \alpha_k^*\alpha_k' e^{-i\omega_k t}\right\}$$
$$\times e^A \begin{pmatrix} e^B & 0\\ 0 & e^{-B}\end{pmatrix}, \tag{23}$$

where

$$A = i\left(\frac{\omega}{2}\right)^2 \sum_{k=1}^{M} \frac{g_k^2}{\omega_k} t - \left(\frac{\omega}{2}\right)^2 \sum_{k=1}^{M} \frac{g_k^2}{\omega_k^2}\left(1 - e^{-i\omega_k t}\right), \tag{24}$$

$$B = \sum_{k=1}^{M} \phi_k\left(\alpha_k^* + \alpha_k'\right) + i\frac{\omega}{2}t, \tag{25}$$

$$\phi_k = \frac{\omega}{2}\frac{g_k}{\omega_k}\left(1 - e^{-i\omega_k t}\right). \tag{26}$$

Here we associate the values α^* with time t and α' with time $t = 0$ as is also evident from (8). The simple form of the last term on the right-hand side of (23) reveals the QND nature of the system-reservoir coupling. Since we are considering the unitary dynamics of the complete Hamiltonian (1) there is no decoherence, and the propagator (23) does not have any off-diagonal terms. In a treatment of the system alone, i.e., an open system analysis of Eq. (1) after the tracing over the reservoir degrees of freedom, it has been shown [25] that the population, i.e., the diagonal elements of the reduced density matrix of the system remain constant in time while the off-diagonal elements that are a signature of the quantum coherences decay due to decoherence, as expected.

Note that though the commonly used coordinate-coupling model describing a free particle in a bosonic bath, explicitly solved by Hakim and Ambegaokar [12], with

$$H = \frac{P^2}{2} + \frac{1}{2}\sum_{j=1}^{M}\left(p_j^2 + \omega_j^2(q_j - Q)^2\right), \tag{27}$$

is seemingly not of the QND type, it can be shown to be unitarily equivalent to a Hamiltonian of the QND type as follows:

$$U_2U_1HU_1^\dagger U_2^\dagger = \frac{P^2}{2} + P\sum_{j=1}^{M}\omega_j q_i + \frac{1}{2}\sum_{j=1}^{M}\left(p_j^2+\omega_j^2 q_j^2\right) + \frac{1}{2}\left(\sum_{j=1}^{M}\omega_j q_j\right)^2, \tag{28}$$

where U_1 and U_2 are the unitary operators

$$U_1 = \exp\left[\frac{i\pi}{2\hbar}\sum_{j=1}^{M}\left(\frac{p_j^2}{2\omega_j}+\frac{1}{2}\omega_j q_j^2\right)\right], \tag{29}$$

$$U_2 = \exp\left[\frac{-i}{\hbar}Q\sum_{j=1}^{M}\omega_j q_j\right]. \tag{30}$$

The above Hamiltonian (28) is of the QND type with $[H_S, H_{SR}] = [P^2/2, P\sum_{j=1}^{M}\omega_j q_j] = 0$. It is commonly known as the velocity-coupling model [29].

2.1. An external mode in resonance with the atomic transition

In this subsection we consider a Hamiltonian which is a variant of the one in (1):

$$H_2 = \frac{\hbar\omega}{2}\sigma_z + \hbar\Omega a^\dagger a - \frac{\hbar\Omega}{2}\sigma_z + \sum_{k=1}^{M}\hbar\omega_k b_k^\dagger b_k + \frac{\hbar\omega}{2}\sum_{k=1}^{M} g_k(b_k+b_k^\dagger)\sigma_z. \tag{31}$$

Here

$$\Omega = 2\vec{\epsilon}.\vec{d}^*, \tag{32}$$

where $\vec{d}$ is the dipole transition matrix element and $\vec{\epsilon}$ comes from the field strength of the external driving mode $\vec{E}_L(t)$ such that

$$\vec{E}_L(t) = \vec{\epsilon}e^{-i\omega t} + \vec{\epsilon}^* e^{i\omega t}. \tag{33}$$

Here we have used the form $-\frac{\Omega}{2}\sigma_z$, associated with the external mode, instead of the usual form $-\frac{\Omega}{2}\sigma_x$ and (31) is of a QND type. Proceeding as in section 2 and introducing the symbol ν^* for the Bargmann representation of the external mode $a^\dagger$ we have the amplitudes of the propagator for (31) in matrix form as

$$u_2 = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11}\end{pmatrix} = \exp\left\{\sum_{k=1}^{M}\alpha_k^*\alpha_k' e^{-i\omega_k t}\right\} \times \exp\left\{\nu^*\nu' e^{-i\Omega t}\right\} e^A \begin{pmatrix} e^{B_2} & 0 \\ 0 & e^{-B_2}\end{pmatrix}, \tag{34}$$

where A is as in Eq. (24),

$$B_2 = \sum_{k=1}^{M}\phi_k\left(\alpha_k^*+\alpha_k'\right) + i\left(\frac{\omega-\Omega}{2}\right)t, \tag{35}$$

and ϕ_k is as in Eq. (26).

2.2. Non-QND spin-Bose problem

In this subsection we consider a Hamiltonian that is a variant of the spin-Bose problem [22, 27, 23]. This addresses a number of problems of importance such as the interaction of the electromagnetic field modes with a two-level atom [30, 31]. Another variant of the spin-Bose problem has been used for treating problems of phase transitions [32, 33] and also to the tunnelling through a barrier in a potential well [34]. Our Hamiltonian is

$$\begin{aligned} H_3 = & \frac{\hbar\omega}{2}\sigma_z + \sum_{k=1}^{M} \hbar\omega_k b_k^\dagger b_k \\ & + \frac{\hbar\omega}{2}\sum_{k=1}^{M} g_k(b_k + b_k^\dagger)\sigma_x. \end{aligned} \tag{36}$$

This could describe, for example, the interaction of M modes of the electromagnetic field with a two-level atom via a dipole interaction. This has a form similar to Eq. (1) except that here the system-environment coupling is via σ_x rather than σ_z. This makes the Hamiltonian (36) a non-QND variant of the Hamiltonian (1). We proceed as in Section II with H_{N_1} (17) now given by

$$H_{N_1} = \frac{\hbar\omega}{2}\left(\gamma^*\frac{\partial}{\partial\gamma^*} - \beta^*\frac{\partial}{\partial\beta^*}\right) + i\frac{\hbar\omega}{2}f(t)\left(\gamma^*\frac{\partial}{\partial\beta^*} + \beta^*\frac{\partial}{\partial\gamma^*}\right). \tag{37}$$

The propagator for H_{N_1} (37) has the same form as N_1 (20) but with Q now satisfying the equation

$$\frac{\partial}{\partial t}Q = \frac{i\omega}{2}\sigma_z Q + \frac{\omega}{2}f(t)\sigma_x Q, \tag{38}$$

with $Q_{ij}(0) = \delta_{ij}, Q(0) = I$. This is solved recursively to yield the series solution

$$\begin{aligned} Q(t) \quad &= \sum_{n=0}^{\infty} Q^{(n)}(t), \\ Q^{(n)}(t) &= \left(\frac{i\omega}{2}\sigma_z\right)^n \int_0^t d\tau_n \int_0^{\tau_n} d\tau_{n-1} ... \int_0^{\tau_2} d\tau_1 \\ & \quad \times \exp\left[\frac{\omega}{2}\sigma_x\left(\int_0^{\tau_1} - \int_{\tau_1}^{\tau_2} + ... + (-1)^n \int_{\tau_n}^{t}\right) d\tau f(\tau)\right]. \end{aligned} \tag{39}$$

Using Eq. (39) and proceeding as before, we obtain the amplitudes of the propagator for the Hamiltonian (36) in matrix form as

$$\begin{aligned} u_3 = & \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix} \\ = & \exp\left\{\sum_{k=1}^{M} \alpha_k^* \alpha_k' e^{-i\omega_k t}\right\} \\ & \times \sum_{n=0}^{\infty} \left(\frac{i\omega}{2}\right)^n \int_0^t d\tau_n \int_0^{\tau_n} d\tau_{n-1} ... \int_0^{\tau_2} d\tau_1 \exp\left\{\kappa^{(n)}\right\} \\ & \times \begin{pmatrix} \cosh\left(\chi^{(n)}\right) & \sinh\left(\chi^{(n)}\right) \\ (-1)^n \sinh\left(\chi^{(n)}\right) & (-1)^n \cosh\left(\chi^{(n)}\right) \end{pmatrix}, \end{aligned} \tag{40}$$

where

$$\kappa^{(n)} = -\left(\frac{\omega}{2}\right)^2 \sum_{k=1}^{M} \frac{g_k^2}{\omega_k^2} \Bigg[(2n+1) - i\omega_k t + (-1)^{n+1} e^{-i\omega_k t} - 2\sum_{l=1}^{n}(-1)^{l+1} e^{-i\omega_k \tau_l} + 2(-1)^n \sum_{l=1}^{n}(-1)^{l+1} e^{-i\omega_k (t-\tau_l)} + 4\sum_{p=2}^{n}\sum_{q=1}^{p-1}(-1)^{p+q} e^{-i\omega_k(\tau_p-\tau_q)}\Bigg], \quad (41)$$

and

$$\chi^{(n)} = -\frac{\omega}{2}\sum_{k=1}^{M}\frac{g_k}{\omega_k}\Bigg[\left(\alpha_k' + (-1)^n \alpha_k^*\right)\left(1 + (-1)^{n+1} e^{-i\omega_k t}\right) + 2\alpha_k^* \sum_{l=1}^{n}(-1)^{l+1} e^{-i\omega_k(t-\tau_l)} - 2\alpha_k' \sum_{l=1}^{n}(-1)^{l+1} e^{-i\omega_k \tau_l}\Bigg]. \quad (42)$$

This agrees with the results obtained in [22, 27]. The matrix on the right-hand side of Eq. (40) contains diagonal as well as off-diagonal terms in contrast to the matrix on the right-hand side of Eq. (23) in which only diagonal elements are present. This is due to the non-QND nature of the system-bath interaction of the Hamiltonian described by Eq. (36) whose propagator is given by Eq. (40), whereas Eq. (23) is the propagator of the Hamiltonian given by Eq. (1) where the system-bath interaction is of the QND type. The simpler form of the structure of the propagator (23) compared to the non-QND propagator (40) reflects on the simplification in the dynamics due to the QND nature of the coupling.

3. Bath of spins

Now we consider the case where the reservoir is composed of spin-half or two-level systems, as has been dealt with by Shao and collaborators in the context of QND systems [18] and also quantum computation [35], and for a nanomagnet coupled to nuclear and paramagnetic spins [19]. The total Hamiltonian is taken as

$$\begin{aligned} H_4 &= H_S + H_R + H_{SR} \\ &= \frac{\hbar\omega}{2} S_z + \sum_{k=1}^{M} \hbar\omega_k \sigma_{zk} + \frac{\hbar\omega}{2}\sum_{k=1}^{M} c_k \sigma_{xk} S_z. \end{aligned} \quad (43)$$

Here we use S_z for the system and σ_{zk}, σ_{xk} for the bath. Since $[H_S, H_{SR}] = 0$, we have a QND Hamiltonian. In the Bargmann representation, we associate the variable β^* with the spin-down state and the variable γ^* with the spin-up state for the bath variables, and we have

$$\begin{aligned} \sigma_z &\longrightarrow \gamma^* \frac{\partial}{\partial \gamma^*} - \beta^* \frac{\partial}{\partial \beta^*}, \\ \sigma_x &\longrightarrow \gamma^* \frac{\partial}{\partial \beta^*} + \beta^* \frac{\partial}{\partial \gamma^*}. \end{aligned} \quad (44)$$

Similarly, the bosonization of the system variable gives

$$S_z \longrightarrow \xi^* \frac{\partial}{\partial \xi^*} - \theta^* \frac{\partial}{\partial \theta^*}, \quad (45)$$

where the variable θ^* is associated with the spin-down state and the variable ξ^* with the spin-up state. The bosonized form of the Hamiltonian (43) is given by

$$H_{B_4} = \frac{\hbar\omega}{2}\left(\xi^*\frac{\partial}{\partial\xi^*} - \theta^*\frac{\partial}{\partial\theta^*}\right) + \sum_{k=1}^{M}\hbar\omega_k\left(\gamma_k^*\frac{\partial}{\partial\gamma_k^*} - \beta_k^*\frac{\partial}{\partial\beta_k^*}\right) + \frac{\hbar\omega}{2}\sum_{k=1}^{M}c_k\left(\gamma_k^*\frac{\partial}{\partial\beta_k^*} + \beta_k^*\frac{\partial}{\partial\gamma_k^*}\right)\left(\xi^*\frac{\partial}{\partial\xi^*} - \theta^*\frac{\partial}{\partial\theta^*}\right). \quad (46)$$

A particular solution of the Schrödinger equation for the bosonized Hamiltonian (46) is obtained by attaching amplitudes to the polynomial parts in the products

$$U_4 = (\theta^* + \xi^*)(\theta' + \xi')\prod_{k=1}^{M}(\beta_k^* + \gamma_k^*)(\beta_k' + \gamma_k'). \quad (47)$$

The initial state for the expanded propagator associated with the bosonized Hamiltonian (46) is

$$U(t=0) = \exp\{\theta^*\theta' + \xi^*\xi'\}\prod_{k=1}^{M}\exp\{\beta_k^*\beta_k' + \gamma_k^*\gamma_k'\}. \quad (48)$$

Using (8), the propagator for the bosonized Hamiltonian (46) is

$$\begin{aligned} u_4(\theta^*, \xi^*, \boldsymbol{\beta^*}, \boldsymbol{\gamma^*}, t; \theta', \xi', \boldsymbol{\beta'}, \boldsymbol{\gamma'}, 0) = &\prod_{k=1}^{M}\int \mathbf{D}^2\{\theta\}\mathbf{D}^2\{\xi\}\mathbf{D}^2\{\beta_k\}\mathbf{D}^2\{\gamma_k\} \\ &\times \exp\Bigg\{\sum_{0\le\tau<t}\Big[\theta^*(\tau+)\theta(\tau) + \xi^*(\tau+)\xi(\tau) \\ &+ \beta_k^*(\tau+)\beta_k(\tau) + \gamma_k^*(\tau+)\gamma_k(\tau)\Big] \\ &- i\frac{\omega}{2}\int_0^t d\tau\Big[\xi^*(\tau+)\xi(\tau) - \theta^*(\tau+)\theta(\tau)\Big] \\ &- i\int_0^t d\tau\omega_k\Big[\gamma_k^*(\tau+)\gamma_k(\tau) - \beta_k^*(\tau+)\beta_k(\tau)\Big] \\ &- i\frac{\omega}{2}\int_0^t d\tau c_k\Big[\gamma_k^*(\tau+)\beta_k(\tau) + \beta_k^*(\tau+)\gamma_k(\tau)\Big] \\ &\times\Big[\xi^*(\tau+)\xi(\tau) - \theta^*(\tau+)\theta(\tau)\Big]\Bigg\}. \end{aligned} \quad (49)$$

On the left-hand side of Eq. (49), $\boldsymbol{\beta^*}$, $\boldsymbol{\gamma^*}$ are vectors with components $\{\beta_k\}$ and $\{\gamma_k\}$, respectively. Now we introduce a complex auxiliary field $f(\tau)$ to decouple the interaction term in (49) as

$$\exp\left(-i\frac{\omega}{2}\int_0^t d\tau c_k\left[\gamma_k^*(\tau+)\beta_k(\tau) + \beta_k^*(\tau+)\gamma_k(\tau)\right]\left[\xi^*(\tau+)\xi(\tau) - \theta^*(\tau+)\theta(\tau)\right]\right)$$

$$= \int D^2\{f\}\exp\left[-i\int_0^t d\tau f^*(\tau)c_k\left(\gamma_k^*(\tau+)\beta_k(\tau) + \beta_k^*(\tau+)\gamma_k(\tau)\right)\right]$$

$$\times \exp\left[\int_0^t d\tau f(\tau)\frac{\omega}{2}\left(\xi^*(\tau+)\xi(\tau)-\theta^*(\tau+)\theta(\tau)\right)\right]. \tag{50}$$

Using (50) in (49) the propagator for the bosonized Hamiltonian (46) becomes

$$\begin{aligned} u_4\left(\theta^*,\xi^*,\boldsymbol{\beta^*},\boldsymbol{\gamma^*},t;\theta',\xi',\boldsymbol{\beta'},\boldsymbol{\gamma'},0\right) \;=\; & \prod_{k=1}^{M}\int D^2\{f\}M_1\left(\theta^*,\xi^*,t;\theta',\xi',0;[f]\right) \\ & \times N_{1_k}\left(\beta_k^*,\gamma_k^*,t;\beta_k',\gamma_k',0;[f^*]\right), \end{aligned} \tag{51}$$

where M_1 is the propagator for

$$H_{M_1}=\frac{\hbar\omega}{2}\left(\xi^*\frac{\partial}{\partial\xi^*}-\theta^*\frac{\partial}{\partial\theta^*}\right)+\frac{i\hbar\omega}{2}f(t)\left(\xi^*\frac{\partial}{\partial\xi^*}-\theta^*\frac{\partial}{\partial\theta^*}\right), \tag{52}$$

and N_{1_k} is the propagator for

$$H_{N_{1k}}=\hbar\omega_k\left(\gamma_k^*\frac{\partial}{\partial\gamma_k^*}-\beta_k^*\frac{\partial}{\partial\beta_k^*}\right)+\hbar f^*(t)c_k\left(\gamma_k^*\frac{\partial}{\partial\beta_k^*}+\beta_k^*\frac{\partial}{\partial\gamma_k^*}\right). \tag{53}$$

Here the propagator M_1 is

$$M_1=\sum_{p=1}^{\infty}\frac{1}{p!}\left[\left(\theta^*,\xi^*\right)\widetilde{Q}\begin{pmatrix}\theta'\\ \xi'\end{pmatrix}\right]^p, \tag{54}$$

where $\widetilde{Q}$ is given by

$$\widetilde{Q}(t)=\exp\left(\frac{i\omega}{2}S_z t-\frac{\omega}{2}S_z\int_0^t d\tau f(\tau)\right), \tag{55}$$

with $\widetilde{Q}_{ij}(0)=\delta_{ij},\widetilde{Q}(0)=I$.

The propagator N_{1_k} is

$$N_{1_k}=\sum_{l=0}^{\infty}\frac{1}{l!}\left[\left(\beta_k^*,\gamma_k^*\right)Q^k\begin{pmatrix}\beta_k'\\ \gamma_k'\end{pmatrix}\right]^l, \tag{56}$$

where Q^k satisfies the equation

$$\frac{\partial}{\partial t}Q^k=i\left(\omega_k\sigma_{z_k}-f^*(t)c_k\sigma_{x_k}\right)Q^{(k)}. \tag{57}$$

This equation can be solved recursively to give

$$Q^k(t)=\sum_{n=0}^{\infty}Q^{k(n)}(t), \tag{58}$$

with

$$Q^{k(0)}(0)=I,\quad Q^{k(n)}(0)(n\neq 0)=0, \tag{59}$$

$$\begin{aligned} Q^{k(n)}(t) \;=\; & \left(i\omega_k\sigma_{z_k}\right)^n\int_0^t d\tau_n\int_0^{\tau_n}d\tau_{n-1}...\int_0^{\tau_2}d\tau_1 \\ & \times\exp\left(-i\sigma_{x_k}c_k\left(\int_0^{\tau_1}-\int_{\tau_1}^{\tau_2}+...+(-1)^n\int_{\tau_n}^{t}\right)d\tau f^*(\tau)\right). \end{aligned} \tag{60}$$

Using Eqs. (54), (56) with $p = 1$, $l = 1$, respectively, in Eq. (51) and making use of Eqs. (55), (58), (59), (60) along with the δ-functional identity (13), the amplitudes of the propagator for the Hamiltonian (43) are obtained in matrix form (in the Hilbert space of H_R) as

$$u_4 = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix} = \prod_{k=1}^{M} \sum_{n=0}^{\infty} (i\omega_k)^n \int_0^t d\tau_n \int_0^{\tau_n} d\tau_{n-1} ... \int_0^{\tau_2} d\tau_1 \times e^{i\frac{\omega}{2}S_z t} \begin{pmatrix} \cos(\Theta^{k(n)}) & i\sin(\Theta^{k(n)}) \\ (-1)^n i \sin(\Theta^{k(n)}) & (-1)^n \cos(\Theta^{k(n)}) \end{pmatrix}, \quad (61)$$

where

$$\Theta^{k(n)} = \frac{\omega}{2} S_z c_k A_n, \quad (62)$$

$$A_n = \sum_{j=1}^{n} (-1)^{j+1} 2\tau_j + (-1)^n t. \quad (63)$$

Now if we expand the terms containing S_z, i.e., make an expansion in the system space, in Eq. (61) we get terms such as

$$e^{i\frac{\omega}{2}S_z t} \cos(\Theta^{k(n)}) = \cos(\frac{\omega}{2} c_k A_n) \begin{pmatrix} e^{i\frac{\omega}{2}t} & o \\ 0 & e^{-i\frac{\omega}{2}t} \end{pmatrix}. \quad (64)$$

Here we have used the fact that

$$e^{S_z A} = \begin{pmatrix} e^{A} & o \\ 0 & e^{-A} \end{pmatrix}. \quad (65)$$

Similarly,

$$e^{i\frac{\omega}{2}S_z t} i \sin(\Theta^{k(n)}) = i \sin(\frac{\omega}{2} c_k A_n) \begin{pmatrix} e^{i\frac{\omega}{2}t} & o \\ 0 & -e^{-i\frac{\omega}{2}t} \end{pmatrix}. \quad (66)$$

The above equations have only diagonal elements. We can see from the above equations that there are 16 amplitudes of the propagator for each mode k of the reservoir out of which only the energy-conserving terms are present due to the QND nature of the system-reservoir coupling.

4. Discussions

We look closely at the forms of the propagators (23) and (61) of the QND type Hamiltonians (1) and (43), respectively. In the first case with an oscillator bath, Eq. (23) involves the matrix

$$\begin{pmatrix} e^{B} & 0 \\ 0 & e^{-B} \end{pmatrix},$$

where B is given by Eq. (25). This can be used to generate the following transformation in phase space:

$$\begin{pmatrix} X \\ P \end{pmatrix} = \begin{pmatrix} e^{B} & 0 \\ 0 & e^{-B} \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}. \quad (67)$$

It can be easily seen from Eq. (67) that the Jacobian of the transformation is unity and it is a phase space area-preserving transformation. The first matrix on the right-hand side of (67) has the form of a 'squeezing' operation [36], which is an area-preserving (in phase space) canonical transformation coming out as an artifact of homogeneous linear canonical transformations [37].

In the second case of a spin bath, Eq. (61) involves the matrix

$$R \equiv \begin{pmatrix} \cos\Theta^{k(n)} & i\sin\Theta^{k(n)} \\ (-1)^n i \sin\Theta^{k(n)} & (-1)^n \cos\Theta^{k(n)} \end{pmatrix}, \quad (68)$$

where $\Theta^{k(n)}$ is given by Eq. (62). For particular n and k, we write $\Theta^{k(n)}$ as Θ. For n even, the above matrix (68) becomes

$$\begin{pmatrix} \cos\Theta & i\sin\Theta \\ i\sin\Theta & \cos\Theta \end{pmatrix} = e^{i\Theta\sigma_x}. \tag{69}$$

Using the Campbell-Baker-Hausdorff identity [38] this matrix can be shown to transform the spin vector $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ as

$$e^{i\Theta\sigma_x} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix} e^{-i\Theta\sigma_x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\Theta & -\sin 2\Theta \\ 0 & \sin 2\Theta & \cos 2\Theta \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}, \tag{70}$$

i.e., the abstract spin vector is 'rotated' about the x-axis by an angle 2Θ. For n odd, (68) becomes (again writing $\Theta^{k(n)}$ for particular n and k as Θ)

$$\begin{pmatrix} \cos\Theta & i\sin\Theta \\ -i\sin\Theta & -\cos\Theta \end{pmatrix} = \sigma_z \begin{pmatrix} \cos\Theta & i\sin\Theta \\ i\sin\Theta & \cos\Theta \end{pmatrix} = \sigma_z e^{i\Theta\sigma_x}. \tag{71}$$

Thus the n-odd matrix is related to the n-even matrix by the spin-flipping energy. The above matrix transforms the spin vector σ as

$$\sigma_z e^{i\Theta\sigma_x} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_y \end{pmatrix} e^{-i\Theta\sigma_x}\sigma_z = e^{i\pi} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\Theta & \sin 2\Theta \\ 0 & \sin 2\Theta & -\cos 2\Theta \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}. \tag{72}$$

It can be easily seen from the right-hand side of the Eq. (72) that the determinant of the transformation of the spin vectors brought about by the n-odd matrix (71) has the value unity. It is well known that the determinant of a rotation matrix is unity [39]. Thus we see that the above transformation has the form of a rotation. Specifically, it can be seen that

$$\sigma_z \begin{pmatrix} \cos 2\Theta & \sin 2\Theta \\ \sin 2\Theta & -\cos 2\Theta \end{pmatrix} = \begin{pmatrix} \cos 2\Theta & \sin 2\Theta \\ -\sin 2\Theta & \cos 2\Theta \end{pmatrix}, \tag{73}$$

and

$$\begin{pmatrix} \cos 2\Theta & \sin 2\Theta \\ -\sin 2\Theta & \cos 2\Theta \end{pmatrix}^T = \begin{pmatrix} \cos 2\Theta & -\sin 2\Theta \\ \sin 2\Theta & \cos 2\Theta \end{pmatrix}. \tag{74}$$

Here T stands for the transpose operation. From the above it is seen that the matrix (68) has the form of the operation of 'rotation', which is also a phase space area-preserving canonical transformation [36] and comes out as an artifact of homogeneous linear canonical transformations [37]. Any element of the group of homogeneous linear canonical transformations can be written as a product of a unitary and a positive transformation [40, 41], which in turn can be shown to have unitary representations (in the Fock space) of rotation and squeezing operations, respectively [37]. It is interesting that the propagators for the Hamiltonians given by Eqs. (1) and (43), one involving a two-level system coupled to a bath of harmonic oscillators and the other with a bath of two-level systems, are analogous to the squeezing and rotation operations, respectively.

5. Conclusions

In this paper we have investigated the forms of the propagators of some QND Hamiltonians commonly used in the literature, for example, for the study of decoherence in quantum computers. We have evaluated the propagators using the functional integral treatment relying on coherent state path integration. We have treated the cases of a two-level system interacting with a bosonic bath of harmonic oscillators (section 2), and a spin bath of two-level systems (section 3). In each case the system-bath interaction is taken to be of the QND type, i.e., the Hamiltonian of the system commutes with the Hamiltonian describing the system-bath interaction. We have shown the commonly occuring free-particle coordinate coupling model to be unitarily equivalent to the free-particle velocity coupling model which is of the QND type. For the variants of the model in section 2, we have examined (a) the case where the two-level system in addition to interacting with the bosonic bath of harmonic oscillators is also acted upon by an external mode in resonance with the atomic transition (section 2.1), and (b) the non-QND spin-Bose problem (section 2.2), which could be used to describe the spin-Bose problem of the interaction of a two-level atom with the electromagnetic field modes in a cavity via a dipole interaction.

The evaluation of the exact propagators of these many body systems could, apart from their technical relevance, also shed some light onto the structure of QND systems. We have found an interesting analogue of the propagators of these many-body Hamiltonians to squeezing and to rotation, for the bosonic and spin baths, respectively. Every homogeneous linear canonical transformation can be factored into the rotation and squeezing operations and these cannot in general be mapped from one to the other – just as one cannot in general map a spin bath to an oscillator bath (or vice versa) – but together they span the class of homogeneous linear canonical transformations and are 'universal'. Squeezing and rotation, being artifacts of homogeneous linear canonical transformations, are both phase-space area-preserving transformations, and thus this implies a curious analogy between the energy-preserving QND Hamiltonians and the homogeneous linear canonical transformations. This insight into the structure of the QND systems would hopefully lead to future studies into this domain.

Acknowledgments

It is a pleasure to acknowledge useful discussions with Joachim Kupsch. The School of Physical Sciences, Jawaharlal Nehru University, is supported by the University Grants Commission, India, under a Departmental Research Support scheme.

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